# TECHNICAL UNIVERSITY OF MOMBASA Faculty of Applied \& Health 

## Sciences

DEPARTMENT OF MATHEMATICS \& PHYSICS<br>UNIVERSITY EXAMINATION FOR DEGREE OF:<br>BACHELOR OF SCIENCE IN CIVIL ENGINEERING BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING BACHELOR OF SCIENCE IN ELECTRICAL \& ELECTRONIC ENGINEERING (BSCE/BSME/BSEE)

SMA 2272/AMA 4203: STATISTICS

## END OF SEMESTER EXAMINATION

SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

## Instructions to Candidates:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consist of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown
This paper consists of THREE printed pages

## Question One (Compulsory)

a) Define the following terms:
(i) Sample space (1 mark)
(ii) Probability mass function (3 marks)
(iii) Independent events
b) A fair coin is tossed three times let " H " represent heads and " T " represent tails:
(i) Determine the sample space
(ii) Let X be the random variable representing the number of heads observed. Find the probability mass function of X
(iii) Determine the cumulative distribution function of X marks)
c) For a radioactive atom, the time between emissions of the particles, $T$, is a random variable which follows the exponential distribution:

$$
f(t)=\left\{\begin{array}{cc}
\lambda e^{-\lambda t} & t>0 \\
0 & \text { elsewhere }
\end{array}\right.
$$

Find the half-life (i.e. the median) of $T$
d) In a normal distribution with mean 40 and variance 4, a random variable $Y$ has a value 42
(i) Determine its standard value
(ii) State whether the value $y=50$ is from this distribution
e) Determine the value of C so that the following function can serve as a probability mass function of the r.v.x

$$
\begin{array}{r}
f(x)=c\left(x^{2}+4\right) \\
\quad \text { for } x=0,1,2,3 \tag{2marks}
\end{array}
$$

f) Suppose the error in the reaction temperature in ${ }^{\circ} \mathrm{C}$, for a controlled laboratory experiment is a continuous random variable having the density function:

$$
f x=\left\{\begin{array}{cc}
\frac{x^{2}}{3} & -1<x<2 \\
0 & \text { elsewhere }
\end{array}\right.
$$

$$
f(x)
$$

(i) Verify that is a pdf
(3 marks)
(ii) Find $\mathrm{P}(0<\mathrm{x}<1)$
g) A manufacturer of power meters claims that only $8 \%$ of the meters will be defective. A vendor had just received a shipment of 20 meters from the manufacturer. The vendor wants to test:

$$
H_{o}: P=0.08 \quad H_{1}: P>0.08
$$ where P is the true proportion of power meters that are $x \geq 3$

defective. Let X be the number of defectives in a sample. Use as the rejection region. Determine the probability of the vendor rejecting the shipment
(3 marks)

## Question Two

Electric fuses normally rated at 30 amps are tested by passing a gradually increasing electric current through them and recording the current, X , at which they blow. The results of this test on a sample of 125 such fuses is shown in the table below:

| Current, X(A) | $25-28$ | $28-29$ | $29-30$ | $30-31$ | $31-32$ | $32-33$ | $33-34$ | $34-35$ | $35-40$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of fuses | 6 | 12 | 27 | 30 | 18 | 14 | 9 | 4 | 5 |

a) Draw a histogram to represent this data
b) Calculate to 2 decimal places
$\begin{array}{ll}\text { (i) The modal current } & \text { (4 marks) } \\ \text { (ii) The mean current } & \text { (5 marks) }\end{array}$
(iii) The standard deviation of the current marks)

## Question Three

a) Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 are summarized as follows:

|  | Shock Resistance |  |  |  |
| :---: | :--- | :---: | :---: | :---: |
| Scratch Resistance |  | High | Low | Total |
|  | High | 40 | 9 | 79 |
|  | Low | 16 | 5 | 21 |
|  | Total | 86 | 14 | 100 |

Let A denote the event that a plastic has high shock resistance and let B denote the event that a disk has high scratch resistance. Determine the following:
(i) $\mathrm{P}(\mathrm{A})$
(1 mark)
(ii) $\mathrm{P}(\mathrm{B})$
(1 mark)
(iii) $\mathrm{P}(\mathrm{A} / \mathrm{B})$ mark)
(iv) $\mathrm{P}(\mathrm{B} / \mathrm{A})$
(1 mark)
(v) Whether events A and B are independent
b) The thermal conductivity of a certain brand of iron box was measured using a new method of measuring thermal conductivity at $100^{\circ} \mathrm{C}$. The following readings in $\mathrm{W} /{ }^{\circ} \mathrm{C}$ were obtained $41.48,42.34$, 41.95, 41.86, 41.18, 41.72, 42.26, 41.81, 42.04. Determine:
(i) A point estimate of thermal conductivity at $100^{\circ} \mathrm{C}$
(ii) The standard error of the point estimate
(iii) The $95 \%$ confidence interval for the thermal conductivity of the iron box at $100^{\circ} \mathrm{C}$
(5 marks)

## Question Four

In an experiment to measure the stiffness of a spring the length of the spring under different loads was measured as follows:

| X (loads) (gms) | 3 | 5 | 6 | 9 | 10 | 12 | 15 | 20 | 22 | 28 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Y(length) (mm) | 10 | 12 | 15 | 18 | 20 | 22 | 27 | 30 | 32 | 34 |

a) (i) Fin the product-moment correlation between X and Y
(8 marks)
(ii) Find the coefficient of determination comment on it
(2 marks)
b) (i) Find the regression equation of length on load
(9 marks)
(ii) Predict the length of the spring when the load is 31grams

## Question Five

a) The weekly number of accidents occurring at a particular stretch of a highway is known to have an average of 5 . Find the probability that:
(i) Less than 4 accidents will occur at the stretch in a week
(ii) No accident occurs at the stretch in a 2-week interval
b) The full volume of an automate filling machine used for filling cans for juice producing firm is normally distributed with a mean of 500 ml and a standard deviation of 10 ml .
(i) Find the probability that the volume is more than 520 ml
(2 marks)
(ii) If specifications require that cans with less than 485 ml and more than 525 ml must be refilled, find the percentage of cans refilled
(iii)In order to provide more customer satisfaction, the firm adopts a new policy to offer cans with higher volume of contents than the market standard of 500 ml while maintaining the standard deviation at 10 ml . The firm then makes adjustments to the filling machine to effect this policy. if a sample of 30 cans from the production line is found to have a mean of 505 ml , test at $5 \%$ level if the new policy is being implemented
( 6 marks)

