THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF ENGINEERING IN MECHANICAL/BUILDING & CIVIL ENGINEERING

SMA 2272: STATISTICS

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: MAY/JUNE 2012

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination
- *Answer Booklet*

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are clearly shown
This paper consists of **FOUR** printed pages

Question 1 (Compulsory - 30 Marks)

a) List the elements of each of the following sample spaces:
   (i) The set of integers between 1 and 70 divisible by 9 (2 marks)
   $$S = \{x \mid x^2 - 5x + 6 = 0\}$$
   (ii) The set (2 marks)

b) An urn contains 3 black balls and 5 white balls. Two balls are drawn in turn with replacement. Determine the probability of

   (i) 2 black balls (2 marks)
   (ii) 1 black and 1 white ball (2 marks)

c) Determine the probabilities in (b) if the balls are drawn without replacement (4 marks)

d) A telephone call occurs at random in the interval (0, t). Let T be its time of occurrence. $0 \le t_0 \le t_1 \le t$:

Determine, where

(i) $P(t_0 \le T \le t_1)$ (1 mark)

(ii) $P(t_o \le T \le t_1 \mid T > T_o)$ (1 mark)

e) (i) Define the term random variable (1 mark)

(ii) A coin is tossed 3 times. Obtain the probability distribution of the number of heads observed. (4 marks)

f) Determine the value of c so that the following function can serve as a probability distribution function of the random variable X:

$$f(x) = c\binom{2}{x}\binom{3}{3-x}, \quad for \ x = 0,1,2$$

(2 marks)

g) The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function.

$$f(x) = \begin{cases} \dfrac{20\ 000}{(x+100)^3}, & x > 0 \\ 0 & elsewhere \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of

(i) At least 200 days
(ii) Anywhere from 80 to 120 days (2 marks)

h) A study of carbon monoxide levels at the Island side of the Likoni Ferry crossing revealed the following data (in parts per million) for 8 days during afternoon drive-time.

CARBON MONOXIDE

1.53	1.50	1.37	1.51	1.55	1.42	1.41	1.48

Determine:

(i) The mean (2 marks)
(ii) Standard deviation (2 marks)

Question 2 (20 Marks

a) A random variable X is exponentially distributed if it has a density function of the form

$$f(x)=\begin{cases} a\,e^{-ax} & x \ge 0 \\ 0, & elsewhere \end{cases}$$

where a > 0

Find an expression for F(x)

b) Sketch
(i) The probability density function , pdf (4 marks)
(ii) The probability distribution function, PDF, of X (4 marks)

c) Determine
(i) $P(0 < X \le 1)$ (3 marks)
(ii) $P(X \ge 4)$ (2 marks)
(iii) The mean of the distribution (3 marks)

Question 3 (20 Marks)

a) It has been observed over time that 70% of engineering graduates from Kenyan universities get absorbed in the local market. Let Y be the number of engineering graduates from Kenyan universities absorbed locally. Of a sample of 25 engineering graduates, find:

(i) $P(Y = 10)$ (3 marks)
(ii) $P(Y \le 3)$ (3 marks)
(iii) The mean μ and standard deviation σ (2 marks)
(iv) Interpret the results in (iii) (1 mark)

b) Based on extensive testing it is determined that the time Y in years before a major repair is required for a particular brand of refrigerator is characterized by the density function

$$f(x)=\begin{cases} \frac{1}{4}\,e^{\frac{-y}{4}} & y \ge 0 \\ 0, & elsewhere \end{cases}$$

(i) If the product is considered a bargain when it takes more than 6 years to require a major repair, determine whether this brand of refrigerator qualifies as a bargain. (4 marks)

(ii) Determine the probability that a major repair is required in the first year. (2 marks)

(iii) Determine the variance of the repair time (5 marks)

Question 4 (20 Marks)

a) A random variable has a gamma distribution with $\alpha = 2 \text{ and } \beta = 1$. Find: (4 marks)

$$P(1.8 < X \leq 2.4)$$

b) A manufacturing company knows from past experience that the relative frequency distribution of the length of time, X in months, between major customer complaints can be modeled by a gamma density function with $\alpha = 3 \text{ and } \beta = 4.18$ months after the firm tightened its qualify control requirements, the first complaint arrived. Does this suggest the mean time between customer complaints may have increased? (9 marks)

c) A coin is tossed 400 times. Use the normal approximation to the binomial to find the probability

i) between 185 and 210 heads
ii) Less than 176 or more than 227 heads (7 marks)

Question 5 (20 Marks)

The data in table 1 show the diameters in mm of a sample of 52 bearings used in motor assembly.

Construct a frequency distribution table for the data starting with the classes 30.0-1.5, 31.5-33.0 etc and use to determine:

(i) The mean and
(ii) The standard deviation for the data (20 marks)

41.0	36.9	37.1	44.8	36.8	30.0	37.2	42.1	36.7	32.7	37.3	41.2	36.6
36.5	33.2	37.4	37.5	33.6	40.5	36.5	37.6	33.9	40.2	36.4	37.7	37.7
34.2	36.2	37.9	36.0	37.9	35.9	38.2	38.3	35.7	35.6	35.1	38.5	39.0
34.8	38.6	39.4	35.3	34.4	38.8	39.7	36.3	36.8	35.5	36.4	40.5	36.6

Table 1