TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE/TECHNOLOGY IN INFORMATION TECHNOLOGY/

SMA 2101/SMA 2172: CALCULUS I

END OF SEMESTER EXAMINATION
SERIES: APRIL 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Differentiate $y = 5x^2 - 3x + 2$ from first principles **(5 marks)**

b) Given that $f(x) = 2x - 1$ and $g(x) = 3 - 5x$ find $fog(x)$ and $fog(x)^{-1}$ **(5 marks)**

c) Find the domain and the range of $y = \sqrt{(x-3)(x+5)}$ **(4 marks)**

d) Find $\frac{dy}{dx}$ given that $y = \sin 3x + \cos 2x$ **(3 marks)**

e) Evaluate the following limits:

(i) $\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$ **(2 marks)**

(ii) $\lim_{x \to \infty} \frac{x^3 + 2x - 1}{6x^2}$ **(2 marks)**

f) Examine the continuity of $f(x) = |x^2 - 5x + 6|$ at x = 3 **(4 marks)**

g) Investigate whether $f(x) = \begin{cases} x^2, & x < 1 \\ 4 - 3x & if x \geq 1 \end{cases}$ is differentiable at x = 1 **(3 marks)**

h) Find $\frac{dy}{dx}$ given that $u = x^2$ and $y = \cos u$ **(2marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Differentiate the following functions:

(i) $y = \frac{3x - 2}{\sqrt{2x + 1}}$ **(3 marks)**

(ii) $y = e^{-t}\left(t^2 - 2t + 2\right)$ **(3 marks)**

(iii) $y = \sec\theta \ \tan\theta$ **(3 marks)**

b) Find and classify the critical points of the curve $y = x^3 - 6x^2 + 9x - 8$ **(8 marks)**

Question Three

a) Define continuity of a function f at a point x = a **(3 marks)**

b) A gas is escaping from a spherical balloon at the rate of $2ft^3/min$. How fast is the surface area shrinking when the radius is 12ft. **(5 marks)**

c) If $y^2 + x^2 = 2y\sqrt{1+x^2}$, show that: $\frac{dy}{dx} = \frac{1}{\sqrt{2}-2}$ at (1, 1) **(5 marks)**

d) Differentiate by first principles $\sqrt{2x+1}$ **(7 marks)**

Question Four

a) A curve is defined parametrically by:

$$y = \frac{2t}{1+t}, \quad x = \frac{1-t^2}{1+t^2}$$

Find its gradient at t = 1 **(7 marks)**

b) Find $f \bullet g \bullet h(x)$ given that $f(x) = \sqrt{x-1}$ $g(x) = x^2 + 2$ and $h(x) = x + 3$. Hence find the range of $f \bullet g \bullet h(x)$

(6 marks)

c) Show that the normal to the curve $3y = 6t - 5t^3$, draw the point $\left(1, \frac{1}{3}\right)$ passes through the origin.
(7 marks)

Question Five

a) Find the integrals of the following functions:

(i) $\frac{2}{x\sqrt{x}}$ **(2 marks)**

(ii) $\frac{1}{x} + \sin x$ **(2 marks)**

(iii) $(x+1)(x+2)$ **(2 marks)**

(iv) $(x-6)^2$ **(1 mark)**

b) Find the area enclosed by the curve $y = 3x^2 + 2$ the x-axis and the lines at x = 3 and x = 5.
(5 marks)

c) A particle P moves in a straight line AB. Its distance x, from A at the end of t seconds is given by $x = 2t^3 - 15t^2 + 36t + 20$. Prove that the velocity of P becomes zero at two points C and D in AB and it acceleration becomes zero at one point E at a time midway between times of arrival at C and D.
(18 marks)