THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

(A Centre of Excellence)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE
BACHELOR OF ENGINEERING IN ELECTRICAL & ELECTRONICS ENGINEERING/ MECHANICAL & AUTOMOTIVE ENGINEERING/INFORMATION TECHNOLOGY/BUIDLING & CIVIL ENGINEERINGCIVIL ENGINEERING

SMA 2101/2172/AMA 4101: CALCULUS I

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2012
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Find the gradient of the tangent and normal at point (2,3) to the hyperbola xy = 6. **(5 marks)**

b) A spherical balloon is inflated at the rate of $2cm^3/s$. Find the rate of growth of the radius if r = 2cm, correct to two decimal places. **(4 marks)**

c) Given functions $f(x)$ and $g(x)$. Show by use of first principle that:

$$(fg)'x = f'(x)g(x) + f(x)g'(x)$$

(5 marks)

d) If function $f : x \to 1 + x - 6/x$ and $g : x \to 1/x$, where $x \neq 0,$ find $h = f(g(x))$ **(2 marks)**

e) Using first principle differentiate: $y = \dfrac{1}{\sqrt{x^2 - 1}}$ **(5 marks)**

f) Find:

(i) $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x - 2}$ **(3 marks)**

(ii) $\lim\limits_{x \to \infty} \dfrac{5x + 1}{10 + 2x}$ **(3 marks)**

g) Find $\dfrac{dy}{dx}$, of $x^2 + 2xy + y^3 =$ at (1, 1) **(3 marks)**

Question Two

a) A projectile is aimed vertically and its height after t seconds, is S metres, where:

$$s = 25.2t - 4.9t^2$$

Find:

(i) Its height and velocity after 3 seconds. **(3 marks)**

(ii) When it is momentarily at rest **(2 marks)**

(iii) Maximum height attained **(3 marks)**

(iv) Acceleration at t = 4 seconds. **(2 marks)**

b) Find the greatest or least value of y on the curve $y = 4x - x^2$ and sketch the curve. **(10 marks)**

Question Three

a) A metal sheet has measurements 8 by 5 metres. Equal squares of side x metres are removed from each corner and the edges are then turned up to make an open box of volume Vm^3.

$$V = 40x - 26x^2 + 4x^3$$

Show that:

Find the maximum possible volume and the corresponding value of x. **(7 marks)**

b) By applying the concept of small changes as used in calculus. Find the approximate value of $\sqrt[3]{1005}$

(5 marks)

c) Show that $g(x) = \frac{2x-1}{x-3}$ can be expressed in the form $g(x) = \frac{a}{x-3} + b$. Find a and b if they are real numbers. **(4 marks)**

d) Define the terms:

(i) Domain **(2 marks)**

(ii) Composite function **(2 marks)**

Question Four

a) (i) Find A in terms of x if:

$$\frac{dA}{dx} = \frac{(3x+1)(x^2-1)}{x^5}$$

(3 marks)

(ii) Give the value of A if x = 2 **(2 marks)**

b) (i) Find the area enclosed by the x-axis, $x = 1,\ x = 3$ and the curve $y = x^3$ **(4 marks)**

(ii) The volume of a cube is increasing at the rate of 2cm^3/s. Find the rate of change of the base when its length is 3cm. **(4 marks)**

c) Find the gradient of the curve:

$x = \frac{t}{1+t},\ y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ **(7 marks)**

Question Five

d) Differentiate:

(i) $y = \frac{\sin x}{1+\cos x}$ **(4 marks)**

$$y^2 = \frac{\tan x}{1 + \tan^2 x}$$

(ii) **(4 marks)**

c) Find:

$$y = 2x^3 + 3x^2 - 12x + 7$$

(i) The turning points of the graph . **(6 marks)**

(ii) Distinguish between maximum and minimum value of the points. **(4 marks)**

(iii) Show that the graph passes through (1, 0) and find the other point. **(2 marks)**