



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENG., MECHANICAL & CIVIL ENGINEERING (YI, SEM I)

SMA 2170: ALGEBRA

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: OCTOBER 2011

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer booklet

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

This paper consist of **THREE** printed pages

- a) i) In how many ways can the letters of the word TROTting be arranged?

$$\frac{21!}{8!13!}$$

- ii) Simplify

[3 marks]

$$\frac{x^{-\frac{1}{2}}(x-1)^{\frac{1}{2}} + x^{\frac{1}{2}}(x-1)^{-\frac{1}{2}}}{x^{\frac{1}{2}}} = \frac{2x-1}{x(x-1)^{\frac{1}{2}}}$$

- b) Show that

[5marks]

$$\log_{10} \frac{a^2 b^3}{100\sqrt{c}}$$

- c) Express in terms of $\log_{10} a$, $\log_{10} b$ and $\log_{10} c$,

[3 marks]

d) If α and β are the roots of the equation $3x^2 + 2 = 8x$, find the values of $\alpha^2 + \beta^2$ and $\alpha^{-1} + \beta^{-1}$ [5 marks]

e) In the expansion of $(1 - 2x + ax^2)^4$ as a series of powers of x , the coefficient of x^3 is zero. Find a and the coefficient of x^4 . [5 marks]

f) By first writing z_1 and z_2 in polar form, find $z_1 z_2$ given that $z_1 = 1 + i$; $z_2 = \sqrt{3} - i$ [5 marks]

(h) Change $6e^{2-i3}$ into $(a+ib)$ form. [4marks]

QUESTION TWO [20 marks]

a) Solve the equation $3 \cdot \sqrt{\frac{x}{x-1}} + 6 \cdot \sqrt{\frac{x-1}{x}} = 11$ [6 marks]

b) Prove that $\log_q p = \frac{1}{\log_p q}$, hence show that $\log_a N \cdot \log_b M = \log_b N \cdot \log_a M$ [7 marks]

c) Express $z = -16$ in polar form hence find all the solutions to $\sqrt[4]{-16}$ [7marks]

QUESTION THREE [20 marks]

(a) Given $ax^2 + bx + c = 0$, derive the quadratic formulae. [5marks]

$$\frac{-25}{2} \left(\frac{1+i2}{3+i4} - \frac{2-i5}{-i} \right)$$

$$= 57 + i24$$

(b) Show that: [6marks]

(c) Prove by induction that:

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

[9marks]

QUESTION FOUR [20 marks]

$$\frac{\sqrt[3]{(1-3x)\sqrt{(1+x)}}}{\left(1+\frac{x}{2}\right)^3}$$

a) Simplify: _____, given that powers of x above the first may be neglected.

[6 marks]

(b) The following estimate is received for printing copies of a pamphlet.

No. of copies	50	100	200	500
Cost in £	11.50	12.50	14.50	20.50

i) Obtain a law giving the cost, £y of x copies graphically. [11marks]

ii) Estimate the cost of 350 copies. [3 marks]

QUESTION FIVE [20marks]

a) If _____ find the values of a and b where a and b are

$$\frac{\sqrt{12}}{\sqrt{6}-\sqrt{2}} - \frac{\sqrt{12}}{\sqrt{6}+\sqrt{2}} = a\sqrt{6} + b\sqrt{2}$$

rational numbers

[4 marks]

b) Evaluate $(1-i)^{100}$ in Cartesian form giving your result simplified as much as possible

[5

marks]

c) Find the value of λ for which the equation $x^2 - x + 1 = \lambda(x^2 + x + 1)$ where $\lambda \neq 1$, has

equal roots and find, also the

Range of values of λ for which the roots are real and unequal.

[6 marks]

d) Use Mathematical Induction to prove that $n^3 - n$ is divisible by 3 whenever n is a positive

integer

[5 marks]