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THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING/BUILDING & CIVIL ENGINEERING
YEAR 3, SEM I

SMA 2370: CALCULUS IV

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: FEBRUARY/MARCH 2012
TIME: 2HOURS

Instructions to Candidates:

You should have the following for this examination

- *Answer booklet*

This paper consists of **FIVE** questions
Answer Question **ONE** (**Compulsory)** from **SECTION A** and any other **TWO** questions from **SECTION B**
Maximum marks for each part of a question are clearly shown
This paper consists of **TWO** printed pages

SECTION A (Compulsory)

QUESTION ONE (30 MARKS)

a) State the first mean value theorem for a function of two variables and verify the theorem for $f(x,y)=x^2y$ at the point (5, 2) (5 marks)

b) Given $\vec{A}=x^2y\,\hat{i}-2xz\,\hat{j}+2yz\,\hat{k}$, find the curl of curl $\vec{A}$. (3 marks)

c) Find a linear expression in x to approximate $f(x)=\sqrt[3]{x}$ in the neighborhood of 8. Use this to find the approximate value of $\sqrt[3]{8.1}$ correct to 4 decimal places. (5 marks)

d) A surveyor estimates the area of a triangular plot of land using the formula $A=\frac{1}{2}ab\sin C$ where *a* and *b* are the lengths of the two sides and C is the included angle.

If the sides are measured to an accuracy of 2% and the angle C measured as 45^{o} is measured to with 1%, calculate approximately, the percentage error in *A*. (6 marks)

e) Show how to transform the improper integral of the second kind $\int_1^2 \frac{dx}{\sqrt{x(2-x)}}$ into a proper integral (4 marks)

f) Given the force $\vec{F}=xy\hat{i}-y^2\hat{j},$ find the work done by $\vec{F}$ from (0,0) to (2,1) along the following paths:-

(i) The straight line $y=\frac{1}{2}x$

(ii) The broken line $x=0,\quad y=1$

Hence determine whether or not $\vec{F}$ is conservative. (7 marks)

SECTION B (Attempt any TWO questions)

QUESTION TWO (20 MARKS)

a) Find the point on the plane $2x-3y+5z=19$ that is nearest to the origin, using the method of Lagrange multipliers. Hence determine the distance of the plane from the origin. (9 marks)

b) Obtain the Fourier sine series of period 2π which represents f on the interval $0,\pi$ where $f(x)=\begin{cases}\pi, & 0\le x\le \frac{\pi}{2}\\ 0, & \frac{\pi}{2}<x\le\pi\end{cases}$

(11 marks)

QUESTION THREE (20 MARKS)

a) Verify the divergence theorem for the sphere $x^2+y^2+z^2=a^2$ if $\vec{F}=x\hat{i}+y\hat{j}+2\hat{k}$ (10 marks)

$$\oint_C (2xy - x^2)dx + (x + y^2)dy$$

b) Verify Green's theorem in the plane for where C is the closed curve of the region bounded by $y = x^2$ and $y^2 = x$ (10 marks)

QUESTION FOUR (20 MARKS

a) State Stoke's theorem (1 mark)

b) Verify Stoke's theorem for the vector field $\vec{F} = x\hat{i} + x\hat{j} + 2xy\hat{k}$ using the hemisphere $x^2 + y^2 + z^2 = 4, z \le 0$

(19 marks)

QUESTION FIVE (20 MARKS)

a) Evaluate $\oint_S \int \vec{A}.\hat{n}\,ds$ where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5. (9 marks)If $z = (x + y)^2$ where $x = r\cos\theta$ and $y = r\sin\theta$ show that $2r^2\frac{\partial^2 z}{\partial r^2} + \frac{\partial^2 z}{\partial \theta^2} = 4r^2$ (11 marks)