



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN CIVIL ENGINEERING, ELECTRICAL & ELECTRONIC ENGINEERING, MECHANICAL ENGINEERING, BUILDING & CONSTRUCTION & BACHELOR OF ENGINEERING IN ELECTRICAL & ELECTRONIC ENGINEERING

SMA 2172: CALCULUS I

AMA 4103: CALCULUS I

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: OCTOBER 2011

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer booklet

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

This paper consist of **THREE** printed pages

Question One (30 marks)

a) Define the following terms:

- (i) A function (2 marks)
- (ii) A limit of a function (2 marks)

$$f(x) = x^2 + x - 1 \quad h(x) = x^2 - x$$

b) If

$$\text{and} \\ f \circ h(x)$$

Find (i) (2 marks)

$$h \circ f(x)$$

(ii) (2 marks)

$$\frac{dy}{dx}$$

c) Find for the following functions:-

$$y = \sin^3 2x$$

(i) (2 marks)

$$x = 2t + 3, y = t^2 - 1 \text{ at } t = 6$$

(ii) (2 marks)

$$\lim_{x \rightarrow \infty} \frac{(2x-3)(3x+5)(4x-6)}{3x^3 + x - 1}$$

d) Examine the limit of (4 marks)

e) Which of the following two functions grows faster $y = 3^x$ or $y = 2^x$ as $x \rightarrow \infty$. Explain. (4 marks)

$$f(x) = 2x + 3$$

f) Find the derivative of by the first principle. (3 marks)

$$\lim_{x \rightarrow 3} (2x + 1) = 7$$

g) Prove that the limit (4 marks)

$$\int \frac{x dx}{x^2 + a^2}$$

h) Evaluate the following integral (3 marks)

Question Two (20 marks)

a) Define the continuity of a function at a point $x = x_0$ (4 marks)

$$x^2 + xy + y = 7$$

b) Find the tangent to the curve at the point (1,2) (4 marks)

c) Newton's Law of universal gravitation states that, the force between any particles having masses M_1 kg and M_2 kg, separated by a distance r (m) is an attraction acting along the line

$$F = \frac{GM_1M_2}{r^2},$$

joining the particles and has a magnitude where G is a universal constant having the same value for all pairs of particles. Two asteroids are approaching each other. The first has a mass of 1000kg and the second a mass of 3000kg.

(i) What is the gravitational force between the asteroids when they are 10km apart? (2 marks)

(ii) How is their force changing at that distance? Explain (3 marks)

$$\lim_{x \rightarrow \infty} \left(\frac{x-1}{x+1} \right)^x$$

d) Evaluate (3 marks)

$$f(x) = \frac{x}{x^2 - 4x + 3}$$

e) Find the vertical asymptotes of (4 marks)

Question Three (20 marks)

$$y = x - 2, \quad x = y^2$$

- a) Find the area bounded on the right by the line $y = x - 2$, on the left by the parabola $x = y^2$ and below by the x-axis. (6 marks)
- b) A bacteria population is growing at a rate equal to 10% of its population after 10 days. Its initial size is 10,000 organisms. How many bacteria are present after 10 days and after 30 days (6 marks)

$$f'(x) \quad f(x) = (x^3 + x^2 + 1)(x^{19} + 16)$$

c) Find of the following (3 marks)

$$h(x) = x^2 \sin x$$

d) (i) Find the derivative of (2 marks)

$$g(x) = \frac{x-2}{2x-5}, \quad g^{-1}(x)$$

(ii) Given , find (3 marks)

Question Four (20 marks)

$$\lim_{x \rightarrow 0} \left(\frac{1 - \cos 2x}{x \sin x} \right)$$

a) Find (i) (3 marks)

$$\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 2x}$$

(ii) (3 marks)

$$y = (x^2 + 1)^3 (x^3 - 1)^2 \quad y = \frac{4x^3 + 6}{x - 1}$$

b) (i) Find the derivatives of and (6 marks)

$$y = t^2 - 1 \quad x = 2t + 3$$

(ii) If and

$$\frac{dy}{dx}$$

Find (3 marks)

c) A square sheet tin, which has the measurement of one centimeter on each side, is used to make an open top box by cutting a small square of tin from each corner for the box to have a large volume as possible (5 marks)

Question Five (20 marks)

a) Evaluate

(i) $\int x e^x dx$ (3 marks)

(ii) $\int_0^{\frac{\pi}{2}} x \cos x dx$ (3 marks)

b) A hard-boiled egg at 98°C is put in a sink of 18°C water to cool. After 5 minutes, the egg's temperature is found to be 38°C . Assuming that the water has not warmed appreciably, how much longer will it take the egg to reach 20°C ? (8 marks)

c) Investigate continuity of $f(x)$ at $x = -1$ and $x = 1$ where:

$$f(x) = \begin{cases} 2 - x, & x < 1 \\ x, & -1 \leq x < 1 \\ 4, & x = 1 \\ 4 - x, & x > 1 \end{cases}$$

(6 marks)