



# THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)

(A Centre of Excellence)

## Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN  
MECHANICAL AND AUTOMOTIVE ENGINEERING

SMA 2374: LINEAR AND BOOLEAN ALGEBRA

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2012

TIME: 2 HOURS

### **Instructions to Candidates:**

You should have the following for this examination

- Answer Booklet

This paper consist of **FIVE** questions in **TWO** sections **A & B**

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

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### **SECTION A (COMPULSORY)**

#### **Question One (30 marks)**

a) Define the following terms as used in linear and Boolean Algebra

- i) Simple proposition (2 marks)
- ii) Singular matrix (2 marks)
- iii) A vector (2 marks)

b) Find the parametric equation for the line through the points P(-3, 2, -3) and Q(1, -1, 4) (4 marks)

c) Find the value of a if the following matrix is singular. (4 marks)

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$$A = \begin{bmatrix} 3 & -9 & 2 \\ 1 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

$$A \cup (A' \cap B) = A \cup B$$

d) Show that (4 marks)

$$\vec{A} = ai - 2j - 2k \quad \vec{B} = 6i + 3j + 2k$$

e) Find the value of a if the vectors  $\vec{A}$  and  $\vec{B}$  are perpendicular. (5 marks)

f) (i) Find the determinant of the matrix. (3 marks)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{bmatrix}$$

in which row 1 row 3 are the same.

(ii) What conclusion can you draw from your answer above? (2 marks)

$$A = 2i - 3j + 7k$$

g) Find the direction of vector of  $A$ . (2 marks)

### SECTION B (Answer any TWO questions from this section)

#### Question Two (20 marks)

a) Find the unit vectors that are tangent and normal to curve  $y = \tan^{-1} x$  at  $\left(1, \frac{\pi}{4}\right)$ . (7 marks)

$$\vec{a} = 4i + 3k \quad \vec{b} = -2i + j + 5k \quad |2\vec{a} + 3\vec{b}|$$

b) If  $\vec{a}$  and  $\vec{b}$  find  $|2\vec{a} + 3\vec{b}|$ . (3 marks)

$$3x - 6y - 2z = 7 \quad 2x + y - 2 = 5$$

c) Find the angle between the two planes  $3x - 6y - 2z = 7$  and  $2x + y - 2 = 5$ . (6 marks)

$$\vec{A} = i - 2j - 2k \quad \vec{B} = 6i + 3j + 2k,$$

d) If  $\vec{A}$  and  $\vec{B}$  find the projection vector of A onto B. (4 marks)

#### Question Three (20 marks)

a) Define the following terms:

i) Proposition (2 marks)

ii) Conjecture (2 marks)

$$p \wedge q \rightarrow r \vee s \Rightarrow (P \wedge q) \rightarrow r \vee s$$

b) Construct a truth table for the statement:

**(10 marks)**

$$A - B = A \cap B'$$

c) Show that (6 marks)

**Question Four (20 marks)**

a) Define the following terms as used in linear algebra.

- i) Matrix (1 mark)
- ii) Zero matrix (1 mark)
- iii) Diagonal matrix (1 mark)

b) (i) State the Cramer's rule for a 3x3 matrix. **(4 marks)**

(ii) Using Cramer's rule solve the matrix equation. **(8 marks)**

$$x + 2y + 4z = 4$$

$$2x + z = 3$$

$$3y + z = 2$$

c) Use Gauss-Jordan elimination method to solve the equation.

$$x_1 - x_2 + x_3 = 0$$

$$-x_1 + x_2 - x_3 = 0$$

$$10x_2 + 25x_3 = 90$$

$$20x_1 + 10x_2 = 80$$

**(4 marks)**

**Question Five (20 marks)**

a) Given that  $n(A) = 3$ ,  $n(B) = 2$  and  $n(A \cap B) = 1$ . Find:

i)  $n(A')$  **(2 marks)**

ii)  $n(A' \cap B)$  **(2 marks)**

iii)  $n(A \cup B)$  **(2 marks)**

$$A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

b) Calculate the Eigen values of the matrix A and its corresponding Eigen vectors if **(6 marks)**

$$v = 2j + 3k$$

c) Find the parametric and Cartesian equation for the line through  $P(2, -9, 5)$ , parallel to **(5 marks)**

d) Find a unit vector perpendicular to both  $\vec{A} = 2i + j - k$  and  $\vec{B} = i - j + 2k$  **(3 marks)**