THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of JKUAT)
Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL/ELECTRONICS/CIVIL/MECHANICAL ENGINEERING
(YR I, SEM I)

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: MAY/JUNE 2012

TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination

- *Answer Booklet*

This paper consists of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are clearly shown
This paper consists of **TWO** printed pages

Question 1 (Compulsory - 30 Marks)

a) Derive the differential equation arising from $z = yf(x) + xg(y)$ (5 marks)

b) Show that every curve of the family $xy = a$ is orthogonal to the curve of the family $x^2 - y^2 = b\ [a, b \neq 0]$ (5 marks)

c) Find the direction cosines of the tangent to the conic $ax^2 + by^2 + cz^2 = 1,\ x + yz = 1$ at the point $P(x, y, z)$ (5 marks)

d) Find the general solution of the differential equation $y^2zp - x^2zq = x^2y$ (6 marks)

e) Solve $\frac{\partial^2 u}{\partial x^2} = 12x^2(t+1)$ given that at $x = 0,\ u = \cos 2t\ and\ \frac{\partial u}{\partial x} = \sin t$ (6 marks)

f) Show that the wave equation $u_u = a^2 u_{xx}$ is variable separable (3 marks)

Question 2 (20 Marks)

a) Find the orthogonal trajectories on the sphere $x^2 + y^2 + z^2 = a^2$ of it's intersection with the paraboloid $\frac{xy}{z} = c, c$ being a parameter (12 marks)

b) Consider a curve which is the intersection of the surfaces $F(x, y, z) = 0\ and\ G(x, y, z) = 0$. Prove that $(dx, dy, dz)\alpha\left(\frac{\partial(F,G)}{\partial(y,z)}, \frac{\partial(F,G)}{\partial(z,x)}, \frac{\partial(F,G)}{\partial(x,y)}\right)$ (8 marks)

Question 3 (20 Marks)

a) Find the differential equation arising from $\phi\left(x + y + z, x^2 + y^2 + z^2\right) = 0$ (7 marks)

b) Verify that the differential equation $\left(x^2 z - y^3\right)dx + 3xy^2 dy + x^3 dz = 0$ is integrable (5 marks)

c) Find the integral curves of the equations $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$ by eliminating one of the variables (8 marks)

Question 4 (20 Marks)

a) A bar length 2 metres is fully insulated along it's sides. It is initially at a uniform temperature of 10ºC and at *t = 0* the ends are plunged into ice and maintained at a temperature of 0ºC. Determine an expression for the temperature at a point Pat a distance x from one end at any subsequent time t seconds after *t = 0*.

$$\left[use\ the\ heat\ conducton\ equation \frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}\right]$$

(15 marks)

b) Solve the equation $ap + bq + cz = 0$ (5 marks)

Question 5 (20 Marks)

a) Eliminate the arbitrary constants a,b,c from $z = ax + by + cxy$ (6 marks)

b) Solve by Laplace transform the boundary value problem

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2},\ u(o,t) = 0, u(3,t) = 0, U(x,o) = 10\sin\ 2\pi x - 6\sin\ 4\pi x$$

(10 marks)

c) Solve $\left(D_x^2 + D_x D_y - 6D_y^2\right)z = 0$ (4 marks)