# THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

**(A Constituent College of JKUAT)**

(A Centre of Excellence)

# Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

**BACHELOR OF SCIENCE IN CIVIL/ELECTRICAL & ELECTRONIC ENGINEERING**

(BSCE,BSEE)

SMA 2371: PARTIAL DIFFERENTIAL EQUATION

**END OF SEMESTER EXAMINATION**

**SERIES:** DECEMBER 2012

**TIME:** 2 HOURS

**Instructions to Candidates:**

You should have the following for this examination

- *Answer Booklet*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

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**Question One (Compulsory)**

**a)** Describe the orthogonal trajectories of $y = kx^2,\ k \neq 0$ **(6 marks)**

**b)** Obtain the general solution to the partial differential equation $(y-z)p + (z-x)q = x - y$ **(4 marks)**

 

**c)** Derive the partial differential equations arising from $z = \frac{1}{2}\left(a^2 + 2\right)x^2 + axy + bx + \phi\left(y + ax\right)$ **(6 marks)**

**d)** Show that the sets of parametric equation $x = a\sin u\cos v,\ y = a\sin u\ \sin v,\ z = a\cos u$ and $x = a\frac{\left(1-v^2\right)}{1+v^2}\cos u,\ y = a\frac{\left(1-v^2\right)}{1+v^2}\sin u,\ x = \frac{2av}{1+v^2}$ represent the same surface of a sphere, centre of origin O. **(6 marks)**

**e)** Find the complete solution of

$$\left(D_x^2 + 3DxDy + 2d_y^z\right)z = e^{3x+y} + 12xy$$

**(8 marks)**

**Question Two**

**a)** Find the direction cosines of the space curve defined by the parametric equations.

$x = -0.5s^2,\ y = 0.25s^3,\ z = 1.5s^2$ through $-2,\ 2,\ 6$ **(6 marks)**

**b)** A long rectangular metal plate has its two long sides and the far end at $O^o$ and the base at $100^o$. The width of the plate is 10cm. Find by the method of separation of variables, the steady-state temperature distribution inside the plate. **(14 marks)**

**Question Three**

**a)** Use Laplace Transforms to solve the partial equation:

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = U$$

subject to the initial condition $u\left(x,o\right) = e^{-5x}$ and $u\left(o,t\right) = o, t > 0$ given that $u\left(x,t\right)$ is bounded for $t > 0, x > 0$ **(7 marks)**

**b)** An infinite metal plate covering the first quadrant has the edge along the y-axis held at 0; and the edge along the x-axis, held at :

$$u\left(x,0\right) = \begin{cases} 100^o & ,\ 0 < x < 1 \\ 0^o & ,\ x > 1 \end{cases}$$

Use Fourier transform to find the steady-state temperature distribution as a function of x and y. Assume temperature distribution as function of x and y. Assume temperature of zero as y tends to infinity. **(13 marks)**

**Question Four**

**a)** Solve the system:

$y^1 = 4y_1 - 2y_2$

$y_2^1 = y_1 + y_2$ subject to the initial conditions $y_1(0) = 3$ and $y_2(0) = -1$ **(14 marks)**

**b)** Find the General solution for

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{2\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} = \sin(3x - y)$$

**(6 marks)**

**Question Five**

**a)** Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan \alpha$ of its intersection with the family of planes parallel to z = 0. **(10 marks)**

b) Find the general integral of the partial differential equations $(2xy-1)p + (z - 2x^2)q = 2(x - y^2)$ and also the particular integral which passes through the line x = 1, y = 0 **(10 marks)**