MOMBASA POLYTECHNIC UNIVERSITY COLLEGE
SECOND YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING IN MECHANICAL/ BUILDING AND CIVIL ENGINEERING.
SMA 2272: STATISTICS
DATE: NOVEMBER/DECEMBER 2011
TIME: 2 HOURS
INSTRUCTIONS:
Answer Question ONE and any other TWO Questions.

## QUESTION ONE: (30 MARKS)

(a) List the elements of each of the following sample spaces:
i) the set of integers between 1 and 50 divisible by 8
ii) the set $S=\left\{x \mid x^{2}+4 x-5=0\right\}$
(b) An engineering system has two components $A$ and $B$. The following events describe the states of the components:

A: first component is good; $\bar{A}$ : first component is defective
B: second component is good; $\bar{B}$ : second component is defective
Tests have shown that $\mathrm{P}(\mathrm{A})=0.8, \mathrm{P}(\mathrm{B} \mid \mathrm{A})=0.85, \mathrm{P}(\mathrm{B} \mid \overline{\mathrm{A}} \quad)=0.75$
Determine the probability that:
i) the second component is good
ii) at least one of the components is good
iii) the first component is good given that the second is good
iv) the first component is good given that at most one component is good
(c) State whether the events represented by components A and B in (b) are
(i) independent
(ii) mutually exclusive
(verify your answer)
(d) A telephone call occurs at random in the interval ( $0, \mathrm{t}$ ). Let T be its time of occurrence. Determine, where $0 \leq t_{0} \leq t_{1} \leq t:$
i) $\quad \mathrm{P}\left(\begin{array}{ll}t_{0} \leq T & \leq t_{1}\end{array}\right)$
ii) $\quad \mathrm{P}\left(t_{0} \leq T \quad \leq t_{1} \mid \mathrm{T}>t_{0}\right)$
(e) Determine the value of c so that the following function can serve as a probability distribution function of the random variable X:

$$
\mathrm{f}(\mathrm{x})=\mathrm{c}\binom{2}{x}\binom{3}{3-x}, \text { for } \mathrm{x}=0,1,2
$$

(f) The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$
\mathrm{f}(\mathrm{x})=\left\{\begin{array}{c}
\frac{20000}{(x+100)^{3}}, x>0 \\
0, \text { elsewhere }
\end{array}\right.
$$

Find the probability that a bottle of this medicine will have a shelf life of
i) at least 200 days
ii) anywhere from 80 to 120 days
(g) A study of carbon monoxide levels at the Island side of the Likoni Ferry crossing revealed the following data(in parts per million) for 8 days during afternoon drive-time.

CARBON MONOXIDE

| 1.53 | 1.50 | 1.37 | 1.51 | 1.55 | 1.42 | 1.41 | 1.48 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

i) If it is known that an exposure to a mean of 1.5 ppm or more of carbon monoxide creates the possibility of death by carbon monoxide poisoning, test at $5 \%$ level, the hypothesis that the people on the Island side of Likoni Ferry crossing are at risk of death by carbon monoxide poisoning assuming a normal distribution.
ii) Suppose the standard deviation of the daily carbon monoxide levels during morning drive-time at the location is 1.45 . Determine, which time, morning or afternoon has more variable carbon monoxide levels

## QUESTION TWO(20MARKS)

(a) The pdf of X is shown in fig. 1
i) Determine the value of a
ii) Graph $\mathrm{F}(\mathrm{x})$ approximately
iii) Determine $P(X \geq 2 \mid X \geq 1$ i

fig. 1
(b) The National Science foundation in the U.S. reports that $70 \%$ of graduate students who earn PhD degrees in engineering are foreign nationals. Consider the number Y of foreign students in a random sample of 25 engineering students who recently earned their PhD .
i) Find $\mathrm{P}(\mathrm{Y}=10)$
ii) Find $\mathrm{P}(\mathrm{Y} \leq 3$ i
iii) Find the mean $\mu$ and standard deviation $\sigma$
iv) Interpret the results in (iii)

## QUESTION THREE (20 MARKS)

(a) It is known from previous data, that the length of time in months between customers' complaints about a certain product is a gamma distribution with $\alpha=2$ and $\beta=4$. Changes were made that involved tightening of quality control requirements. Following these changes, it took 20 months before the first complaint. Determine whether the quality control tightening was effective.
(9 marks)
(b) Based on extensive testing it is determined that the time Y in years before a major repair is required for a particular brand of refrigerator is characterized by the density function

$$
f(y)=\left\{\begin{array}{l}
\frac{1}{4} e^{\frac{-y}{4}}, y \geq 0 \\
0, \text { elsewhere }
\end{array}\right.
$$

i) If the product is considered a bargain if takes more than 6 years to require a major repair, determine whether this brand of refrigerator qualifies as a bargain.
(7 marks)
ii) Determine the probability that a major repair is required in the first year.
(4 marks)

## QUESTION FOUR (20 MARKS)

(a) A quality control supervisor in a cooking oil refining factory is interested in the variation, $\sigma^{2}$, of the amount of fill. If $\sigma^{2}$ is large some cans will contain too much and others too little. To estimate the variation of the fill the supervisor randomly selects 10 cans and weighs the contents of each. The weights in kg are listed in table 1. Construct a $90 \%$ confidence interval for the true variation in fill of the cans if the sample is assumed to be from a normal population.
(13marks)

| 7.96 | 7.90 | 7.98 | 8.01 | 7.97 | 7.96 | 8.03 | 8.02 | 8.04 | 8.02 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Table1
(b) A computer hard disk manufacturing firm wishes to evaluate the performance of its hard disk
memories by measuring the average time between failures. To estimate this value, the time between failures for a random sample of 45 hard disks was recorded with a mean of 1762 hours and standard deviation of 215 hours.
i) Estimate the true mean time between failures with 90\% confidence interval
(5marks)
ii) If the hard disk memory system is running properly, the true mean time between failures will exceed 1700 hours. Based on the interval in part (i), determine whetherthe hard disk memory for this firm is running properly (2marks)

## QUESTION FIVE (20MARKS)

The data in table 2 show the number of kilometers travelled by 100 test cars of a certain model on a gallon of fuel.
(a) Construct a frequency distribution table for the data starting with the classes 30.0-31.5, 31.5-33.0, etc.
(10 marks)
(b) Determine the mean for the data
(c) Determine the standard deviation for the data

| 36.3 | 41.0 | 36.9 | 37.1 | 44.8 | 36.8 | 30.0 | 37.2 | 42.1 | 36.7 | 32.7 | 37.3 | 41.2 | 36.6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 32.9 | 36.5 | 33.2 | 37.4 | 37.5 | 33.6 | 40.5 | 36.5 | 37.6 | 33.9 | 40.2 | 36.4 | 37.7 | 37.7 |
| 40.0 | 34.2 | 36.2 | 37.9 | 36.0 | 37.9 | 35.9 | 38.2 | 38.3 | 35.7 | 35.6 | 35.1 | 38.5 | 39.0 |
| 35.5 | 34.8 | 38.6 | 39.4 | 35.3 | 34.4 | 38.8 | 39.7 | 36.3 | 36.8 | 32.5 | 36.4 | 40.5 | 36.6 |
| 36.1 | 38.2 | 38.4 | 39.3 | 41.0 | 31.8 | 37.3 | 33.1 | 37.0 | 37.6 | 37.0 | 38.7 | 39.0 | 35.8 |
| 37.0 | 37.2 | 40.7 | 37.4 | 37.1 | 37.8 | 35.9 | 35.6 | 36.7 | 34.5 | 37.1 | 40.3 | 36.7 | 37.0 |
| 33.9 | 40.1 | 38.0 | 35.2 | 34.8 | 39.5 | 39.9 | 36.9 | 32.9 | 33.8 | 39.8 | 34.0 | 36.8 | 35.0 |
| 38.1 | 36.9 |  |  |  |  |  |  |  |  |  |  |  |  |

Table 2

