# TECHNICAL UNIVERSITY OF MOMBASA Faculty of Applied \& Health 

## Sciences

DEPARTMENT OF MATHEMATICS \& PHYSICS<br>UNIVERSITY EXAMINATION FOR DEGREE OF:<br>BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGY BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS

AMA 4117: PROBABILITY \& STATISTICS<br>END OF SEMESTER EXAMINATION<br>SERIES: APRIL 2015<br>TIME ALLOWED: 2 HOURS

## Instructions to Candidates:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consist of FOUR questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown
This paper consists of THREE printed pages

## Question One (Compulsory)

a) Define the following terms:
(i) Statistic
(ii) Random variable
(iii) Mutually exclusive events
b) A four sided die has the number 1 through 4 written on its side, one on each side. If the die is rolled twice.
(i) Write down suitable sample space S
(2 marks)
(ii) If X is the random variable denoting the sum of numbers appearing, determine the values of x
(2 marks)

$$
P(2 \leq x \leq 5)
$$

(iii) Determine

## marks)

$$
\text { c) If } P(A)=0.4 P(B)=0.6 \quad P(A \cup B)=0.7 \text {. Calculate } P(A \cap B)
$$

d) Define the following terms:
(i) Skewness
(ii) Kurtosis
e) Compute the mean and standard deviation of the following data:

| Observation (x) | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency (y) | 5 | 2 | 13 | 16 | 3 |

f) Find a and b in the regression line

$$
y=a+b x
$$

$$
\sum x=440, \quad \sum y=4400, \quad \sum x y=25607, n=10 \sum x^{2}=25796
$$

given that
(5 marks)

## Question Two

a) The following data was obtained from the manufacturers of electronic calls. A sample of electronic cells was taken and life spans were recorded as shown in the following table:

| Life span (hrs) | $1600-1799$ | $1800-1999$ | $2000-2199$ | $2200-2399$ | $2400-2599$ | $2600-2799$ | $2800-2999$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No of cells | 25 | 32 | 46 | 58 | 40 | 30 | 7 |

Using the assumed mean $\mathrm{A}=2299.5$, find:
(i) Mean life span of cells
(ii) Variance of the life span of cells
(iii) Standard deviation of the life span of the cells marks)
b) Let X be a random variable denoting the life time of an electrical equipment and suppose that the pdf

$$
f(x)=c e^{-c x}
$$

of $x$ is for $x>0$
(i) Determine the constant C
(3 marks)
(ii) Calculate the probability that x is at least equal to 10 time units
(iii) If the probability in part (ii) is 0.5 , what is the given value of C ?

## Question Three

a) Compute the moment coefficient of skewness from the following data and hence comment on the skewness of the frequency curve:
(10 marks)

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ | $50-60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 6 | 12 | 22 | 24 | 16 | 12 |

b) State Bayes theorem
(2 marks)
c) Three machines I, II and III manufacture $30 \%$, $30 \%$ and $40 \%$ respectively of the total output of certain items. Of them $4 \%, 3 \%$ and $2 \%$ respectively are defective. One item is drawn at random from the total output and is tested.
(i) What is the probability that the item is defective
(3 marks)
(ii) If it is found to be defective, what is the probability the item was produced by machine I
(5 marks)

## Question Four

$$
f(x)=3 x^{2}, 0<x<1
$$

a) Let X be a random variable with pdf
(i) Show that $\mathrm{f}(\mathrm{x})$ is indeed a pdf
(ii) Calculate the quantities $\mathrm{E}(\mathrm{X}), \mathrm{E}\left(\mathrm{X}^{2}\right)$ and $\operatorname{Var}(\mathrm{X})$
(iii) If the random variable Y is defined by calculate $\mathrm{E}(\mathrm{Y})$ and the var (Y)
(4 marks)

$$
f(x)=e^{-x}, x>0
$$

b) Given that x is a random variable with p.d.f then: $M_{x}(t)$
(i) Find the mgf of $\mathrm{f}(\mathrm{x})$
(4 marks)

$$
M_{x}(t)
$$

(ii) Using obtain the quantities, $\mathrm{E}(\mathrm{x})$ and var (x)

## Question Five

a) In one stage of development of a new medication for an allergy, an experiment is conducted to study how different dosages of the medication affect the duration of relief from the allergic symptoms. Ten patients are included in the experiments. Each patient receives a specific dosage of the medication and is asked to report back as soon as the protection of the medication seems to wear off. The results were recorded below which shows the dosage ( x ) and respective duration of relief ( y ) for the patients:
(i) Draw a scatter diagram of the data
(4 marks)
(ii) Compute the least squares estimates of $a$ and $b$ and draw the regression line
(iii) Find the coefficient of determination $\mathrm{R}^{2}$ and hence give its interpretation
(4 marks)

| x | 3 | 4 | 5 | 6 | 6 | 7 | 8 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 9 | 5 | 12 | 14 | 16 | 22 | 18 | 24 | 22 |

b) A vendor submits tots of fabric to a textile manufacturer. The manufacturer wants to know if the lot average breaking strength exceeds 200psi. If so she wants to accept the lot. Past experience indicates that a reasonable value of the variance of breaking strength is $100(\mathrm{psi})^{2}$. Is the average breaking strength of the lot exceeding 200psi at $5 \%$ level of significance
(6 marks)

