TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE (BSMC)

BACHELOR OF SCIENCE IN INFORAMTION TECHNOLOGY (BSIT)

AMA 4210/SMA 2230: PROBABAILITY & STATISTICS

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:

(i) Probability generating function **(1 mark)**

(ii) Probability density function **(2 marks)**

(iii) A random variable **(1 mark)**

b) An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.98. Suppose that the classifications are independent and three parts are inspected. Let the random variable X denote the number of parts that are correctly classified, determine the probability mass function of X **(4 marks)**

c) (i) The range of the random variable X is (0, 1, 2, 3, x) where x is unknown. If each value is equally likely and mean of x is 6, determine x **(3 marks)**

(ii) If $h(x) = x^2$, determine $E(h(x))$ **(3 marks)**

d) The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected. **(3 marks)**

e) Let the continuous random variable X denote the diameter of a hole drilled in a sheet metal component. The target diameter is 12.5 milimeter. Most random disturbances to the process result in larger diameters. Historical data show that the distribution of X can be modeled by a probability density function $f\left(x = 20e^{20(x-12.5)}\right), x \geq 12.5$

(i) If a part with a diameter larger than 12.6 is scrapped, what proportion of parts is scrapped. **(2 marks)**

(ii) What proportion of parts is between 12.5 and 12.6 min? **(2 marks)**

f) If the cumulative distribution function of a random variable variable X is given as:

$$f(x) = \begin{cases} 0 & x < 0 \\ 0.2x & 0 \leq x < 4 \\ 0.04x + 0.64 & 4 = x < 9 \\ 1 & 9 \leq x \end{cases}$$

Determine he pdf of x **(3 marks)**

g) Let x denote the time between detection of a particle with a Geiger counter and assume that x has an exponential distribution with $\lambda = 1.4$ minutes. What is the probability that a particle can be detected within 30 seconds of starting the counter. **(3 marks)**

h) Let X be random variable having the geometric distribution with parameter P, determine the probability generating function of x **(3 marks)**

Question Two

a) Let x be a random variable having a binomial distribution with parameter P:

(i) Determine the probability generating function of x **(4 marks)**

(ii) Determine the mean of x **(3 marks)**

(iii) Determine the variance of x **(4 marks)**

b) If x is a random variable show that the variance of x can be given as:

$$\sigma^2 = \sum x^2 f(x) - \mu^2$$

(3 marks)

c) Two dice are tossed and their sum of their outcomes noted. If the random variable x denote the sum of the numbers appreciating:

(i) Determine the probability distribution of x **(3 marks)**

(ii) The mean and variance of x **(4 marks)**

Question Three

a) The phone lines to an airline reservation system are occupied 40% of the time. Assume that the evens the lines are occupied on successive calls are independent and if 10 calls are placed to the airline.

(i) What is the probability that for exactly three calls the lines are occupied. **(3 marks)**

(ii) What is the probability that for at least one call the lines are not occupied **(3 marks)**

(iii) What is the expected number of calls in which the lines are all occupied? **(2 marks)**

(iv) Interpret your answer in (iii) above **(1 mark)**

b) Customers arrive randomly at service point at an average rate of 30 per hour. Assuming that the arrivals follow a poisson process, calculate the probability that:

(i) No customer arrives in any particular minute **(3 marks)**

(ii) Exactly one customers arrives **(2 marks)**

(iii) Two or more customers arrive in any particular minute **(3 marks)**

(iv) Three or few customers arrive in any particular minute **(3 marks)**

Question Four

a) (i) Consider the case where X_1 and X_2 are independent poisson random variables with parameters $\lambda_1 \ and \ \lambda_2$ respectively, find the distribution of the random variable $y = x_1 + x_2$ 1 **(8 marks)**

(ii) What would you conclude for the answer in (i) above? **(2 marks)**

b) If X has density function f_x, and $g(x) = x^2$, then find the distribution function of $Y = g(x) = x^2$

(5 marks)

c) The probability density function of the time failure of an electronic component in a copier (in ours)

$$f(x) = \frac{e^{x/1000}}{1000}$$

for x > 0 determine:

(i) The probability that a component lasts more than 3000 hours before failure. **(2 marks)**

(ii) The number of hours at which 10% of all components have failed. **(3 marks)**

Question Five

a) Aptitude test scores of a job applicants are normally distributed with a mean of 140 and standard deviation of 20.

(i) What is the probability that a score will be in the interval of 100 to 180? **(3 marks)**

(ii) If 500 applicants take the test, how many would you expect to score 145 or below **(4 marks)**

(iii) What proportion of the scores are between 110 and 125 **(3 marks)**

b) Weekly demand for a liquid reagent stocked by a supplier is normally distributed. The mean is 250 gallons and the standard deviation is 80 gallons. How many gallons should be available for a week if the supplier wants to ensure that the probability of running out of stock does not exceed 0.02? **(5 marks)**

c) The manufacturing of semiconductor chips produces 2% defective chips. Assume the chips are independent and that a lot contains 1000 chips:

(i) Approximate the probability that more than 25chips are defective **(2 marks)**

(ii) Approximate the probability that between 20 and 30 chips are defective **(3 marks)**