TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE MATHEMATICS & COMPUTER SCIENCE (BMCS)
BACHELOR OF SCIENCE IN STATISTICS & COMPUTER SCIENCE (BSSC)

AMA 4210: PROBABILITY & STATISTICS II

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:
  (i) Sample space **(1 mark)**
  (ii) A probability density function **(2 marks)**

b) An optical inspection system is to distinguish among different part types. The probability of a correct classification of any part is 0.95 suppose that the classifications are independent and three parts are inspected. let the random variable x denote the number of parts that are correctly classified, determine the probability mass function of x **(3 marks)**

c) The range of the random variable X is (0, 1, 2, 3, x) where X is unknown. If each value is equally likely and the mean of x is 6, determine x **(3 marks)**

If $h(x) = x^3$, determine E (h(x)) **(3 marks)**

d) If you buy a lottery ticket in 50 lotteries in each of which your chance of winning a price is 1/100 what is the probability that you will win a price at least once **(4 marks)**

e) Show that the expectation of a continuous uniform random variable over the interval (α,β) is $\frac{\beta+\alpha}{2}$ (4 marks)

f) If the cumulative distribution function of a random variable x is given as:

$$Fx=\begin{cases}0 & x<0\\ 0.2x & 0\le x<4\\ 0.04x+0.64 & 4\le x<9\\ 1 & 9\le x\end{cases}$$

. Determine the pdf of x **(3 marks)**

g) Suppose the probability density function of the length of a computer cables is $f(x)=0.1$ from 1200 to 1210mm. If the length specifications are 1195 L x 1205mm, what proportion of cables are within specifications **(3 marks)**

1195<x<1205

h) Let X be a random variable having the geometric distribution with parameter P, determine the probability generating function of x **(4 marks)**

Question Two

a) Let X be a random variable having a binomial distribution with parameter P. Determine:
   (i) The moment generating function of X **(4 marks)**
   (ii) Mean of X **(3 marks)**
   (iii) Variance of X **(4 marks)**

b) Let X be exponentially distributed with parameter X. Calculate E(x) **(4 marks)**

c) Two dice are tossed and their sum of their outcomes noted. If the random variable X denote the sum of the numbers appearing, calculate the variance of X **(5 marks)**

Question Three

a) A company employs 800men under the age of 55. Suppose that 30% carry a marker on the male chromosome that indicates an increased risk for highly blood pressure. if 10 men in the company are tested for the market in this chromosome, what is the probability that exactly 1 man has the marker? **(3 marks)**

b) The phone lines to an airline reservation system are occupied 30% of the time. Assume that the events the lines are occupied on successive calls are independent and if the 10 calls are placed to the airline, what is the probability that for:
   (i) Exactly three calls the lines are occupied **(3 marks)**
   (ii) At least one call the lines are not occupied **(3 marks)**
   (iii) What is the expected number of calls in which the lines are all occupied **(2 marks)**

c) Customers arrive randomly at a service point at an average rate of 30 per hour. Assuming that the arrival is following a Poisson process, calculate the probability.

(i) Two or more customers arrive in any particular minute **(3 marks)**

(ii) Three or few customers arrive in any particular minutes **(3 marks)**

Question Four

a) Let X_1 and X_2 be independent Poisson random variables with respective mean X_1 and X_2 show that the distribution of X + Y is a Poisson distribution with mean $X_1 + X_2$ **(8 marks)**

b) (i) Let X be an exponential distribution with a pdf given as $f(x) = \lambda e^{-\lambda x}$. Show that the moment generation function of x given as $\frac{\lambda}{\lambda - t}$ **(4 marks)**

(ii) Determine the variance of X **(4 marks)**

c) Let X be a discrete random variable. Determine the mean of the function ax + b where a and b are constant **(4 marks)**

Question Five

a) Aptitude test scores of a job applicants are normally distributed with a mean of 140 and standard deviation of 20.

(i) What is probability that a score will be in the interval of 100 to 180? **(3 marks)**

(ii) If 500 applicants take the test, how many would you expect to score 14.5 or below **(4 marks)**

(iii) What should be the pass mark if 67% of all the candidates were to pass **(4 marks)**

b) An electronic office product contains 5000 electronic components. Assume that the probability that each component operates without failure during the useful life of the product is 0.999 and assume that the component approximate the probability that 10 or more of the original 500 components fail during the useful life of the product. **(4 marks)**

c) Hits to a high volume website are assumed to follow a Poisson distribution with a mean of 10,000 per day, find an approximate value such that the probability that the number of hits in a day exceed that value is 0.01 **(5 marks)**