TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE (BSMC 12S)

AMA 4215: PROBABILITY & STATISTICS III

END OF SEMESTER EXAMINATION
SERIES: APRIL 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:
(i) Joint probability mass function **(2 marks)**
(ii) Marginal probability mass function **(2 marks)**

b) Let x and y be two random variables with a joint a probability mass function $fxy(x, y)$ show that:

$$E(x) = \sum_R x \quad f_{xy}(x, y)$$

(4 marks)

c) The table below shows the values of the random variable x and y.

x	y	f_{xy} (x,y)
1	1	$1/4$
1.5	2	$1/8$
1.5	3	$1/4$
2.5	4	$1/4$
3	5	$1/8$

(i) Show that the above function is a joint probability mass function. **(1 mark)**

(ii) Determine the conditional probability of x given that y = 2 **(3 marks)**

d) (i) Determine the value of C that makes the function $f(x, y) = ce^{-2x-3y}$ a joint probability density function over the range x > 0 and 0 < y < x **(3 marks)**

(ii) Determine the marginal probability distribution of x. **(3 marks)**

e) If x and y are independent normal random variables with E (x) = 0, V (x) = 4, E (y) = 10 and V(y) = 9. Determine the probability that $2x + 3y < 30$ **(4 marks)**

f) The table below gives a joint probability distribution of x and y:

x	1	1	2	4
y	3	4	5	6
f_{xy} (x,y)	$1/8$	$1/4$	$1/2$	$1/8$

Determine:

(i) Covariance of x and y **(4 marks)**

(ii) Correlation between x and y **(4 marks)**

Question Two

a) Suppose that x is a normal random variable with mean μ and variance δ^2 show that the distribution of $y = (x-\mu)^2 / \delta^2$ is a chi-squared distribution with one degree of freedom. **(10 marks)**

b) Suppose that x has a binomial distribution with probability mass function:

$$fx = \binom{n}{x} px(1-p)^{n-x}, \; x = 0,1....n$$

Determine:

(i) The moment generating function. **(4 marks)**

(ii) The mean and variance of binomial random variable. **(6 marks)**

Question Three

A web site contains 100 pages and 60%, 30% and 10% of the pages contain 1ow, moderate and high graphic content respectively. A sample of four pages is selected without replacement and x and y denote the number of pages with moderate and high graphics output in the sample. Determine:

(i) fxy (x, y) **(5 marks)**

(ii) fx (x) **(3 marks)**

(iii) E(x) **(3 marks)**

(iv) E(Y1X = 3) **(3 marks)**

(v) V(Y1X = 3) **(3 marks)**

(vi) Are x and y independent? **(3 marks)**

Question Four

Table below shows the weights Z to nearest pound, heights x to the nearest inch and ages to the nearest year of 12 boys:

Weight (z)	64	71	53	67	55	77	57	56	51	76	68
Height (x)	57	59	49	62	51	55	48	52	42	61	57
Age (y)	8	10	6	11	8	10	9	10	6	12	9

a) Find the least-squares regression equation of z on x and y. **(10 marks)**

b) Determine the estimated values of z from the given values of x and y. **(5 marks)**

c) Determine the error in estimating the weight of a boy aged 8 and is 57 inches tall. **(3 marks)**

Question Five

a) Let X_1 and X_2 be two independent Poisson random variables with parameters X_1 and X_2 respectively. Find the probability distribution of the random variable $Y_1 = X_1 + X_2$ if $Y_2 = X_2$. **(10 marks)**

b) Let X_1, X_2,….Xn be a random variable with a mean $\bar{x}$ show that:

(i) $E\left(\overline{x}\right) = \mu$ **(3 marks)**

(ii) $V\left(\overline{x}\right) = \frac{\delta^2}{n}$ **(3 marks)**

If the mean and the variance of the population are given as μ and δ^2 respectively.

c) Soft-drink cans are filled by a automated filling machine. The mean fill volume is 12.1 fluid ounces and the standard deviation is 0.1 fluidounce. Assume that the fill volumes of the cans are independent, normal random variables what is the probability that the average volume of 10 cans selected from this process is less than 12 fluids ounces? **(4 marks)**