TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING (BSME 11M)

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

SECTION A (COMPULSORY)

Question One

a) Show that the orthogonal trajectories of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ are $y^2=\frac{4}{\theta}$ where θ is a function of x **(8 marks)**

b) Find the partial differential equation arising from $z=yf(x)+xg(y)$ where f(x) and g(y) are arbitrary functions. **(5 marks)**

c) Find a complete solution of the partial differential equation:

$$(mz-ny)\frac{\delta z}{\delta x}+(nx-1z)\frac{\delta z}{\delta y}=ly-mx$$

(6 marks)

d) Find the direction cosines of the tangent at the point (x, y, z) to the cone $x^2+y^2=z^2\tan^2\alpha,\ z=k,(k=)cons\tan t$

(3 marks)

e) Find the general solution $\left(D_x^2-6D_xD_y+9Dy^2\right)z=\cos(4x-5y)+12xy$ **(8 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Solve the equation:

$$\frac{\delta^2 z}{\delta x^2}+2\frac{\delta^2 z}{\delta x\delta y}-3\frac{\delta^2 z}{\delta y^2}-\frac{\delta z}{\delta x}-7\frac{\delta z}{\delta y}-2z=0$$

(10 marks)

b) Find the integral surface of the linear partial differential equation:

$$x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$$

(10 marks)

Question Three

a) Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curves of intersection with the planes parallel to the plane z = 0 **(10 marks)**

b) Find the general solution of:

$$y_1'=2y,-3y_2$$
$$y_2'=y,+6y^2$$

(10 marks)

Question Four

a) Derive the wave equation for a vibrating string, namely $\frac{\delta^2\mu}{\delta x^2}=\frac{1}{c2}\frac{\delta^2 u}{\delta t^2}$ where $c^2=\frac{\tau}{\rho}$

(7 marks)

b) Solve the wave equation in (a) above satisfying the Cauchy conditions

$$u(o,t)=u(L,t)=0t\geq 0$$
$$u(x,0)=f(x),0\leq x\leq L$$
$$ut/_{t=0}=g(x),0\leq x\leq L$$

where f and g are given functions and L is a given constant. **(13 marks)**

Question Five

a) A metal plate coincides with the square in the xy-plane whose vertices are point s(0, 0), (1, 0), (1, 1) and (0, 1). The two faces of the sheet are insulated and the metal sheet is so thin that heat flow in it can be regarded as two dimensional. Edges parallel to the x-axis are insulated and the left edge manifanied at constant temperature 0°C. If the temperature distribution $u(1, y) = f(y)$ is manifained along the right hand edge, find the steady state temperature distribution throughout the sheet.

(13 marks)

b) Evaluate:

(i) $L\left\{\frac{\delta u}{\delta t}\right\}$ **(3 marks)**

(ii) $L\left\{\frac{\delta^2 u}{\delta t^2}\right\}$ **(4 marks)**