TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2278: ORDINARY DIFFERENTIAL EQUATIONS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

<u>**Instructions to Candidates:**</u>
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Explain what is a homogeneous function, hence determine the homogeneity of the function:

$$f(x, y) = e^{y/x} + \tan^{y/x}$$

(4 marks)

$$F(s) = \frac{4s + 6}{s^2 - 9}$$

b) Find the inverse laplace transform of **(6 marks)**

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c) Using the method of undetermined coefficient determine a general solution of an equation

$$\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 4e^{5x}$$

(7 marks)

d) Use the method of Frobenius to find the solution of the differential equation given:

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$$

(10 marks)

e) Find the general solution of:

$$\frac{dy}{dx} + y = e^{3x}$$

(3 marks)

SECTION B (Answer any TWO questions from this section)

Question Two

a) Find the Laplace transform of:

$$e^{-3t}(2\cos 5t - 3\sin 3t)$$

(4 marks)

$$(5x^2 + 4xy)dx + (2x^2 + 3y)dy = 0$$

b) Solve the equation **(7 marks)**

c) An electric circuit consists of an inductance of 0.1 henry a resistance of 20 ohms and a condenser of capacitance 25 microfarads. Find the charge q and the current i at any time t, given that the initial

$$\frac{dq}{dt} = 0$$

conditions are q = 0.05 coulumbs and i   when t = 0 if:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E(t)$$

(9 marks)

Question Three

$$\frac{dy}{dx} + y\cot x = \cos x$$

$$x = \pi/2 \qquad y = 5/2$$

a) Solve   to obtain the particular solution given that at   and

(6 marks)

b) Obtain a general solution of the equation:

$$\frac{dy}{dx} + \frac{x^2 - xy + y^2}{xy}$$

(8 marks)

$$y\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx}\right)^2 - 2\left(\frac{dy}{dx}\right)$$

c) Solve the second order differential equation **(6 marks)**

Question Four

a) Find the laplace inverse of:
$$\frac{s+2}{s^2-4s+3}$$

(5 marks)

b) Find the particular solution for the initial value problem:

$$(x^2+y^2)dx+2xy\,dy=0 \quad y(1)=1$$
$$\text{if}$$

(8 marks)

$$L\frac{di}{dt}+Ri=E(t),$$

c) Given the equation find an expression for i if E(t) = E₀ when the initial current is i₀ provided that L = 3 henries, R = 15 ohms in 60 cycle sine wave of amplitude 110 volts, while i = 0 when t = 0

(7 marks)

Question Five

a) By separation of variables solve:
$$y\tan x\frac{dy}{dx}=(4+y^2)\sec^2 x$$

(4 marks)

b) A particle of mass 2kg moves along the x-axis attracted towards the origin O by a force whose magnitude is numerically equal to 8x. If it is initially at rest at x = 20 and has also a damping force whose magnitude is numerically equal to 8 times the instantaneous speed. Find the equations of displacement and velocity of the particle at any time t. **(8 marks)**

c) Find the particular solution of:

$$\frac{dy}{dx}=\frac{1+x^2}{1+y^2}$$
$$\text{if } y(0)=3$$

(4 marks)

$$f(x,y)=4x^2-3xy+y^2$$

d) Given determine whether it is homogeneous and indicate the degree of homogeneity. **(4 marks)**