



# TECHNICAL UNIVERSITY OF MOMBASA

## Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

**BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING**

**BACHELOR OF ENGINEERING IN ELECTRICAL & ELECTRONIC ENGINEERING**

**BACHELOR OF SCIENCE IN BUILDING & CIVIL ENGINEERING**

**SMA 2271: ORDINARY DIFFERENTIAL EQUATIONS**

**END OF SEMESTER EXAMINATION**

**SERIES: DECEMBER 2013**

**TIME ALLOWED: 2 HOURS**

### **Instructions to Candidates:**

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Maximum marks for each part of a question are as shown

This paper consists of **THREE** printed pages

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### **Question One (Compulsory)**

a) Consider the equation:

$$\left( y^2 + 2x^2 \frac{dy}{dx} \right) \frac{d^2y}{dx^2} + 2(y+x) \left( \frac{dy}{dx} \right)^2 + x \frac{dy}{dx} + y = 0$$

- (i) Show that the above differential equation is non-linear (3 marks)  
(ii) Find the first integral of the above equation (3 marks)

$$y = (x) = x \sec x$$

- b) Show that the function is a solution of the differential equation

$$y^{11}(\tan x)y^1 - \frac{\tan x}{x} \cdot y = \frac{1}{x^2} \cdot y^3$$

(4 marks)

- c) Solve the IVP by separation of variables method.

$$2x(y+1)dx - ydy = 0; y(0) = -2$$

(4 marks)

- d) Show that the differential equation:

$$(x^2 - xy + y^2)dx - xydy = 0$$

is homogeneous and hence solve it

(4 marks)

- e) Show that the differential equation general solution. (6 marks)

$$(2x^2 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy$$

= 0 is exact and find its

(6 marks)

- f) Solve the total differential equation  $y = 1$

$$(3x^4y - 1)dx + x^5dy = 0$$

by integrating factors method when  $x = 1$ ,

(5 marks)

### Question Two

- a) Solve the equations:

$$y'' - 4y' + y' + 6y = 0$$

(i)

(3 marks)

$$4D^2 - 4D + y = 0$$

(ii)

(3 marks)

- b) Find the general solution of:

$$y''' - 3y'' + 9y' + 13y = 0$$

(4 marks)

$$y''' - y' = 4e^{-x} + 3e^{2x}$$

- c) Solve the equation

$$\text{when } x = 0, y' = -1, y'' = 2$$

by the method of undetermined coefficient with the condition:

(10 marks)

### Question Three

$$L^{-1} \left\{ \frac{5s+1}{s^2 - s - 12} \right\}$$

- a) Determine

(5 marks)

$$\frac{dx}{dt} - 2x = 4$$

- b) Solve the differential equation

given that  $x = 1$  at  $t = 0$  by laplace transterm method

(4 marks)

- c) Show that  $f_1 = \cos x$  and  $f_2 = \sin x$  are linearly independent solutions of the differential equation:  $y'' + y = 0$

(3 marks)

- d) Consider the differential equation  $x^2(1-x)y'' + (1-x)y' + y = 0$  find the singular points of the above differential equation and determine whether they are regular or irregular.

(4 marks)

- e) Show that the equation  $x^3y''' - 6xy' + 12y = 0$  has linearly independent solutions each of the form  $y = x^r$

(4 marks)

#### Question Four

- a) Find a series solution of the differential equation

$$y'' + x^2y' + y = 0$$

about the point  $x = 0$

(10 marks)

- b) Find two independent series solutions of the following Bessel equation of order one by method of Frobenius about the origin

(10 marks)

$$x^2y'' + xy' + (x^2 - 1)y = 0$$

#### Question Five

- a) Briefly explain the difference between the degree and order of a differential equation (2 marks)

- b) Solve the second order differential equation:

$$y \frac{d^2y}{dx^2} = 2 \left( \frac{dy}{dx} \right)^2 - 8 \frac{dy}{dx}$$

(9 marks)

- c) The velocity of a particle moving along the x-axis is proportional to x. At time  $t = 0$ , the particle is located at  $x = 3$  and at time  $t = 12$  seconds it is at  $x = 6$ . Find its position when  $t = 6$ . (9 marks)