TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN CIVIL ENGINEERING
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2371: PARTIAL DIFFERENTIAL EQUATIONS

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

<u>**Instructions to Candidates:**</u>
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

$$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

a) Find the equation **(5 marks)**

$$\phi\left(x + y + z, \quad x^2 + y^2 - z^2\right) = 0$$

b) Eliminate the arbitrary function ϕ from the equation **(6 marks)**

$$y^2 p - xyq = x(z - 2y)$$

c) Find the general solution of the partial differential equation **(5 marks)**

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} + z^2 = 0$$

d) Find the integral surface of which passes through the hyperbola $xy = x + y, \ z = 1$ **(5 marks)**

$$xz^3 dx - zdy + 2ydz = 0$$

e) Test for integrability of the equation   and hence solve it   **(5 marks)**

$$\frac{\partial^2 z}{\partial x} + z = 0$$

$$z = e^y \qquad \frac{\partial z}{\partial x} = 1$$

f) Solve   given that x = 0,   and   **(4 marks)**

Question Two

$$U_{xx} + 2U_{xy} - 3U_{yy} = 0$$

a) Find the characteristics of the equation   **(5 marks)**

b) Reduce the equation to the appropriate canonical form   **(13 marks)**

c) Obtain its general solution   **(2 marks)**

Question Three

a) Using Charpits auxiliary equations, find the complete integral of the differential equation $\left(p^2 + q^2\right)y = qz$

                                                     (15 marks)

$$z = ax^3 + bx^2 y + cxy^2 + \frac{dy^4}{x}$$

b) Eliminate a, b, c and d from   **(5 marks)**

Question Four

a) Find the orthogonal trajectories on the conicoid (x + y) z = 1 of the conics in which its cut by the

$$x - y + z = k$$

    system of planes   where K is a parameter   **(12 marks)**

$$x^2 p^2 + y^2 q^2 = z^2$$

b) Solve   **(5 marks)**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2}$$

c) Solve   **(3 marks)**

Question Five

$$\frac{dx}{x^2\left(y^3 - z^3\right)} = \frac{dy}{y^2\left(z^3 - x^3\right)} = \frac{dz}{z^2\left(x^3 - y^3\right)}$$

a) Find the integral curve of the equation   **(8 marks)**

b) A rod whose surface is measured has a length of 3 units. The end of the rod is kept at 0°C and its

$$u(x,0) = 5\sin 4\pi x - 3sn8\pi x + 2\sin 10\pi x$$

    initial temperature at any point x, 0<x<3 is given by   . Find the
    temperature at any given time t   **(12 marks)**