TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE MATHEMATICS & COMPUTER SCIENCE

AMA 4325: PARTIAL DIFFERENTIAL EQUATIONS I

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) Find The orthogonal trajectories of the one parameter family of curve $x^2 + y^2 = 2cx$ where c is a constant **(8 marks)**

b) Derive the partial differential equation arising from $\phi\left(x^2 + 2yz, x + y + z\right) = 0$ in the form Pp + Qq = R where P, Q, R are functions of x, y, z **(3 marks)**

c) Show that the direction cosines for the tangent at the point (x, y, z) to the conic $ax^2 + byz + cz^2 = 1, \; x + y + z = 1$ are proportional to $\left(by - cz, \; cz - ax, \; ax - by\right)$ **(3 marks)**

d) Change the variables to polar coordinates in the partial differential equation:

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sqrt{x^2 + y^2}$$

Hence or otherwise solve the equation **(9 marks)**

$$r + 2s + 10t = \cos(2x - 3y)$$

e) Solve **(7 marks)**

Question Two

a) Solve by direct integration $\frac{\partial^2 x}{\partial x \partial y} = x^2 y$ and find a particular solution for which $z(x,0) = x^2$ and $z(1, y) = \cos y$

(8 marks)

b) Use the Jacobi method to find a complete integral of the equation $p^2 x + q^2 y = z$ **(12 marks)**

Question Three

a) Solve by Charpits method $q = -xp + p^2$ **(10 marks)**

b) Use Monge's integration method to find a complete solution of the equation:

$$r + 4s + t + rt - s^2 = 2$$

(10 marks)

Question Four

a) Find the orthogonal trajectories of the conicoid $z(x + y) = 4$ of a cone in which it is cut by the system of planes $x - y + z = k$ where k is a parameter **(9 marks)**

b) Solve the heat conduction equation below by the method of separation of variables:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t},\ k =$$

constant subject to the following boundary condition

$$u = u(x,0) = f(x), 0 \le x \le L \quad \left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0,\ t \ge 0$$

(11 marks)

Question Five

a) Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also the particular integral which passes through the line x = 1 and y = 0 **(11 marks)**

b) Classify and express in canonical form the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \left(5 + 2y^2\right)\frac{\partial^2 z}{\partial x \partial y} + \left(1 + y^2\right)\left(4 + y^2\right)\frac{\partial^2 z}{\partial y^2} = 0$ and find the characteristics of the equation

(9 marks)