# TECHNICAL UNIVERSITY OF MOMBASA Faculty of Applied \& Health 

 SciencesDEPARTMENT OF MATHEMATICS \& PHYSICS<br>UNIVERSITY EXAMINATION FOR DEGREE OF:<br>BSSE

## SMA 2472: OPERATIONS RESEARCH <br> END OF SEMESTER EXAMINATION <br> SERIES: APRIL 2015 <br> TIME ALLOWED: 2 HOURS

## Instructions to Candidates:

You should have the following for this examination

- Mathematical tables
- Scientific Calculator

This paper consist of FIVE questions
Answer question ONE (COMPULSORY) and any other TWO questions
Maximum marks for each part of a question are as shown
This paper consists of FOUR printed pages

## Question One (Compulsory)

a) State the definition of operations research according to the operations research society of America
b) A firm produces 3 products. Three products are processed on 3 different machine. The time required to manufacture one unit of each of the three products and the daily capacity of the three machines are given in the table below:

| Machine | Time per unit $(\mathrm{min})$ |  | $\begin{array}{l}\text { Machine Capacity } \\ \text { Min/Day }\end{array}$ |
| :--- | :--- | :--- | :--- | :--- |
|  | Product 1 | Product 2 | Product 3 |$)$

It is required to determine the daily number of units to be manufactured for each product. The profit per unit for product 1,2 and 3 is shs 4,3 and 6 respectively.
Formulate the mathematical L.P model that will maximize daily profit
c) The purchase price of a machine is kshs 52,000 . The installation charge amount to kshs 14400 and its scrap value is only kshs 6400 and maintenance cost in various year is given below:
$\begin{array}{lllllllll}\text { Year } & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \text { Maintenance cost } & 1000 & 3000 & 4000 & 6000 & 8400 & 11600 & 16000 & 19200\end{array}$
After how many years should the machine be replaced? Assume that the machine replacement can be done only at the year ends
(5 marks)
d) Solve by simplex method the following L.P problem

$$
\begin{aligned}
& 3 x_{1}-x_{2}+2 x_{3}=7 \\
& 2 x_{1}+4 x_{2} \geq-12 \\
& -4 x_{1}+3 x_{2}+8 x_{3} \leq 10 \\
& x_{1} x_{2} x_{3} \geq 0
\end{aligned}
$$

e) Consider the network shown below. The three time estimation, the expected activity duration and the variances are shown along the arrows. The earliest expected times and the latest available occurrence times are computed and put along the nodes. Find the probability of completing the project in 12 days
f) The following matrix gives the payoff of different strategies $S_{1} S_{2} S_{3}$ against conditions $N_{1} N_{2} N_{3}$ and $\mathrm{N}_{4}$ Indicate the decisions taken under the following approach

|  | $\mathrm{N}_{1}$ | $\mathrm{~N}_{2}$ | $\mathrm{~N}_{3}$ | $\mathrm{~N}_{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~S}_{1}$ | 4000 | -100 | 6000 | 18000 |
| $\mathrm{~S}_{2}$ | 20000 | 5000 | 400 | 0 |
| $\mathrm{~S}_{3}$ | 20000 | 15000 | -2000 | 1000 |

(i) Pessimistic
(ii) Optimistic
(iii) Regret
marks)

## Question Two

a) Briefly define the operating characteristics of a quelling system
(5 marks)
b) Show that in quelling theory, the expected number of units in the system is given by:

$$
\frac{\lambda}{\mu-\lambda}
$$

c) A person repairing radios fins that the time spent on the radio sets has exponential distribution with mean 20 minutes. if the radios are repaired in the order in which they come and their arrival is approximately Poisson with average rate of 15 for 8 hours day, what is the repairmen expected idle time each day? How many jobs are ahead of the average set just brought in?
d) State any FOUR assumptions of the quelling model

## Question Three

a) Define the following terms as used in network analysis:
(i) Event
(ii) Path
(iii) Dummy marks)
b) A project schedule has the following characteristics:

| Activity | Time (weeks) |
| :--- | :--- |
| $1-2$ | 4 |
| $1-3$ | 1 |
| $2-4$ | 1 |
| $3-4$ | 1 |
| $3-5$ | 6 |
| $4-9$ | 5 |
| $5-6$ | 4 |
| $5-7$ | 8 |
| $6-8$ | 1 |
| $7-8$ | 2 |
| $8-10$ | 5 |
| $9-10$ | 7 |

Required:
(i) Construct the network

## (4 marks)

(ii) Compute E and L for each event
(iii) Find the critical path

## Question Four

a) Define simulation as used in 012
b) Briefly discuss the steps involved in Monte Carlo simulation
c) A dentists schedule all how patience for 30 minutes appointments. Some of the patience take more less than 30 minutes depending on the type of dental work to be done. The following summary shows the various category of work their probabilities and time needed to complete the work.

| Category | Time required (min) | Probability of Category |
| :--- | :--- | :--- |
| Filling | 45 | 0.4 |
| Crown | 60 | 0.15 |
| Cleaning | 15 | 0.15 |
| Extraction | 45 | 0.10 |
| Cleanup | 15 | 0.20 |

Simulate the dentists clinic for four hours and determine the average waiting time for patient as well as the $\qquad$ of the doctor. Assume that all the patients show up at the clinic at exactly their scheduled arrival times starting at 8am. Use the following random numbers for handling the problem
d) Customers arrive at a service facility to get the required service. The inter arrival and service times are constant and are 1.8 minutes and 4 minutes respectively. Simulate the system for 14 minutes
(4 marks)

## Question Five

a) Express the following L.P problem into standard form:

$$
\begin{aligned}
& \text { Maximize } \mathrm{z}=\quad \mathrm{z}=7 x_{1}+5 x_{2} \\
& 2 x_{1}+3 x_{2} \leq 20 \\
& .3 x_{1}+x_{2} \geq 10 \\
& x_{1}, x_{2} \geq 0
\end{aligned}
$$

(4 marks)
b) Interpret the following simplex final table:

|  | $\mathrm{c}_{\mathrm{j}}$ | 2 |  | 3 |  | 4 | 0 |
| ---: | :--- | :---: | :---: | :---: | :---: | :---: | ---: |
| 0 | 0 |  |  |  |  |  |  |
| CB | Basic | $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | b |
| 0 | $\mathrm{~S}_{1}$ | $4 / 3$ | 0 | 0 | 0 | $-3 / 2$ | 60 |
| 3 | $\mathrm{x}_{2}$ | $1 / 3$ | 1 | 0 | 0 | $-1 / 2$ | 60 |
| 4 | $\mathrm{x}_{3}$ | $1 / 3$ | 0 | 1 | 0 | $1 / 2$ | 120 |
|  | $\mathrm{z}_{\mathrm{j}}$ | $7 / 3$ | 3 | 4 | 0 | $1 / 2$ | 660 |
|  | $\mathrm{c}_{\mathrm{j}}-\mathrm{Z}_{\mathrm{j}}$ | $-1 / 3$ | 0 | 0 | 0 | $-1 / 2$ |  |

c) A typist of an office of a company receives on the average 20 letters per day for typing. The typist worker 8 hours a day and it takes on the average 20 minutes to type a letter. The cost of a letter waiting to be mailed is 80 per hour and the cost of the equipment plus salary of the typist is 45 per day.
(i) What is the typist utilization rate
(ii) What is the average number of letters waiting to be typed
d) The yearly cost of two machine A and B when money value is neglected is shown below. Find the cost patters if money value is 101 per year and hence find which machine is more economical:

| Year | 1 | 2 | 3 |
| :--- | :--- | :--- | :--- |
| Machine A | 1800 | 1200 | 1400 |
| Machine B | 2800 | 200 | 1400 |

(6 marks)

