TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF TECHNOLOGY INFORMATION TECHNOLOGY

ICS 2211: NUMBERAL LINEAR ALGEBRA

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **FOUR** printed pages

Question One (Compulsory)

a) Estimate the eigen values of the matrix A using the Gerchgorin bounds:

$$A=\begin{pmatrix}1 & 2 & -1\\ 1 & 1 & 1\\ 1 & 3 & -1\end{pmatrix}$$

(4 marks)

b) Prove that the minimal polynomial m(λ) of a matrix $A(n\times n)$ divides every polynomial that has A as a zero. **(4 marks)**

c) Consider the matrix A

$$A = \begin{pmatrix} \lambda & 2 \\ 2 & \lambda - 3 \end{pmatrix}$$

Find A^{-1} if A is non-singular for what value of λ is the matrix A nonsingular. **(3 marks)**

d) Use Cramer's rule to solve the system of linear equations:

$$-2x + 3y - z = 1$$
$$x + 2y - z = 4$$
$$-2x - y + z = -3$$

(5 marks)

e) Given the second derivative mid-point formula

$$f''(x_o) = \frac{1}{h^2}\{f(x_o - h) - 2f(x_o) + f(x_o + h)\} - \frac{h^2}{12} f^{(4)}(\xi)$$

where $\frac{h^2}{12} f^{(4)}(\xi)$ is the error term and $x_o - h < \xi < x_o + h$ for some ξ approximate $f''(2)$ given that $f(x) = xe^x$

taking h =0. and h = 0.2. Give the errors **(5 marks)**

f) Apply Gaussian elimination to the system

$$E_1 : 0.003000\ x_1 + 59.14x_2 = 59.17$$
$$E_2 = 5.291x_1 - 6.130x_2 = 46.78$$

using partial pivoting and four-digit arithmetic with rounding, compare the results with the exact solution x_1 = 10.0 and x_2 = 1.00 that the differential equation $(2x^2 - xy^2 - 2y + 3)dx - (x^2y + 2x)dy$ is exact and find its general solution. **(6 marks)**

g) Master tobacco finds that it can sell in two distinct markets. If it sells Q_1 units in the first market and Q_2 units in the second market, the revenue functions are:

$$R_1 = A_1Q_1 - B1Q_2$$
$$R_2 = A_2Q_2 - B2Q_2^2$$

and the total cost function is $C = A_3 + B_3(Q_1 + Q_2)$

Given that the A's and B's are known constants (positive) find the quantities Q_1 and Q_2 that will maximize profit. **(5 marks)**

h) Let B and C be inverse of A. Then show that BA = AC = I **(2 marks)**

Question Two

a) Define the following terms as used in linear programming:

(i) Optimization
(ii) Feasible solution
(iii) Constraints

(iv) Objective function **(4 marks)**

b) Broadways produces two types of bread one at a cost of 50 shillings per loaf, the other at a cost of 60 shillings per loaf, Assume that if the first bread is sold at x shillings a loaf and the second at y shillings a loaf, then the number of loaves that can be sold each is given by the formula:

$$N_1 = 250(y - x)$$
$$N_2 = 32000 + 250(x - 2y)$$

Determine x and y for maximum profit **(5 marks)**

c) Maximize graphically max $P = x + 4y$ subject to

$$-x + 2y \leq 6$$
$$5x + 4y \leq 40$$
$$x, y \geq 0$$

(3 marks)

d) Use simplex method in the following maximization problem:

Maximize $P = 2x + 6y + 4z$

subject to

$$2x + 5y + 2z \leq 38$$
$$4x + 2y + 3z \leq 57$$
$$x + 3y + 5z \leq 57$$
$$x, y, z \geq 0$$

(8 marks)

Question Three

a) Prove that a matrix $A_{(n\times n)}$ is diagonalizable if A has n linearly independent eigen vectors **(6 marks)**

b) Using the Jacobi method find all the eigen values and corresponding eigen vectors of the matrix:

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$

given that $\tan 2\theta = \dfrac{2a_{ik}}{a_{ii} - a_{kk}}$

and

$$S1 = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 1 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

, using two rotation **(6 marks)**

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 3 & 2 \\ 3 & .0 & 1 \end{pmatrix}$$

c) Given the matrix

- **(i)** Write the characteristic polynomial **(2 marks)**
- **(ii)** Write the characteristics equation **(1 mar)**
- **(iii)** Find the eigen values **(2 marks)**
- **(iv)** Find the eigen vectors corresponding to each eigen value **(3 marks)**

Question Four

a) The linear system $A_x = b$ given by $\begin{pmatrix} 1 & 2 \\ 1.0001 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 3.0001 \end{pmatrix}$ has the unique solution $x = (1,1)^T$. Determine the residual vector for the poor approximation:

$$\bar{x} = (3, -0.0001)^T$$

(3 marks)

b) Solve the system of equation by using Gaussian elimination:

$$2x_1 + x_2 + x_3 = 1$$
$$2x_1 - 2x_2 + 3x_3 = 4$$
$$2x_1 + 3x_2 - x^3 = 5$$

(5 marks)

c) Find the minimal polynomial of a matrix given by:

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{pmatrix}$$

(6 marks)

d) Show that $A = \begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix}$ is not diagonalizable **(3 marks)**

e) Find the algebraic multiplicity of an eigen value λ for the matrix A **(3 marks)**

$$A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

Question Five

a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ by making use of Cayley-Hamilton theorem

(6 marks)

b) If A and B are non-singular matrices then AB is non-singular, prove that $(AB)^{-1} = B^{-1}A^{-1}$

(2 marks)

c) Form the augment A:I to find the inverse of the matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & .5 & 1 \end{pmatrix}$$

(7 marks)

d) Given that (adjoint A) $A = |A| I$ where I is identify matrix, determine adjoint of the matrix given below, hence determine the inverse of the matrix.

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{pmatrix}$$

(5 marks)