TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING

SMA 2471: NUMERICAL ANALYSIS

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2013
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

$I\left(h, \frac{h}{2}, \frac{h}{4}\right)$ $\int_0^1 \frac{dx}{1+x}$

a) Use Romberg method to compute to determine correct to 4d.p given h = 0.5 if $I(h) = 0.7084,\ I\left(\frac{h}{2}\right) = 0.6970$ and $I\left(\frac{h}{4}\right) = 0.6941$ **(4 marks)**

b) Find length L at T = 372.1k using Newton divided difference method provided the relationship between length L(m) and temperature T(k) on a structure is:

T(Kelvin)	361	367	378	387	399

L (Metres)	154.9	167.0	191.0	212.5	244.2

(5 marks)

c) Use Simpson's rule to find the area under the curve $y = f(x)$ that passes through the three points P_1 (0,2), P_2 (1,3) and P_3(2, 2) **(3 marks)**

d) Find the step size h such that the absolute value of the error for the trapezoidal rule is less than 5 x 10-9 when used to solve $\int_2^7 \frac{dx}{x}$ **(4 marks)**

e) Solve $\frac{dy}{dx} = x - y$ subject to y(o) = 1 where h = 0.1 using Milne's predictor-corrector method by first obtaining the values y_o, y_1, y_2 and y_3 using the Euler's method. **(6 marks)**

f) Use the Runge Kutta 4th order to solve IVP $y' = \frac{t - y}{2}$ at x = 0.25 if y(0) = 1 given h = 0.25 **(4 marks)**

g) Apply the 2 point Gauss-Legendre rule to approximate $\int_{-1}^{1} \frac{dx}{x+2}$ **(4 marks)**

Question Two

a) Determine the step size h to be used in tabulation of $f(x) = \sin x$ in the interval (1, 3) so that a linear interpolation is correct to 4 d.p. **(6 marks)**

b) Using Taylor's method solve $xy' = x - y \quad y(2) = 2$ if $x = 2.1$ at **(5 marks)**

c) Apply the forth order Runge-Kutta method to approximate the solution of the initial value problem

$$\frac{dy}{dt} = \left(\frac{y}{t}\right)^2 + \left(\frac{y}{t}\right)$$

if y(1) = 1 and h = 0.1 at y (1.1) **(9 marks)**

Question Three

a) Solve by Euler's method the differential equation $\frac{dy}{dx} = -y$ given that $y(0) = 1$ and h = 0.01 for n = 3 **(4 marks)**

b) Use the trapezium rule hence the Romberg method for $I\left(h, \frac{h}{2}\right)$ to solve $\int_{4.0}^{5.2} \log_e x\,dx$ if h = 0.4 **(5 marks)**

c) Apply the finite difference tables to determine a polynomial of the lowest degree corresponding to a set of equally spaced values of x if:

x	0	0.5	1.0	1.5	2.0	2.5	3.0
f(x)	0.25	-1.5	-1.75	-0.50	2.25	6.50	12.25

d) Solve $\frac{dy}{dx} = y - x$ where $y(0) = 2$ to find y (0.1) if h = 0.1 using the 4th order Runge-Kutta method. **(6 marks)**

Question Four

a) An alternating current i is measured at equal intervals of 2 milliseconds as follows:

Time (ms)	0	2	4	6	8	10	12
Current (A)	0	3.5	8.2	10.0	7.3	2.0	0

Use Simpson's 1/3 rule to determine the charge q in millicoulombs given by $q = \int_0^{12} idt$ **(3 marks)**

b) Use Taylor's series to find y(0.1) given $y' = x - y^2$ and $y(0) = 1$ correct to 4 d.p **(6 marks)**

c) Find y(0.4) using Adams-bash forth predictor-corrector method if $\frac{dy}{dx} = x + y$ when y(0) = 1 and h = 0.1 by first finding y_0, y_1, y_2, and y_3 using Euler's method **(7 marks)**

d) Determine an approximate value of sin0.15 by Langrange linear interpolation given $\sin 0.1 = 0.09983$ and sin 0.2 = 0.19867 **(4 marks)**

Question Five

a) Find a unique quadratic polynomial of degree 2 or less such that f(0) = 1, f(1) = 3 and f(3) = 55 using the Langrange interpolation **(5 marks)**

b) Use Euler's modified method to determine y (0.05). If $\frac{dy}{dx} = y + x^2 \quad y(0) = 1$ and h = 0.05 **(6 marks)**

c) Evaluate $\int_0^{\frac{\pi}{2}} \sin x dx$ using trapezoidal rule with n = 10 **(5 marks)**

d) Determine the volume of a solid generated by revolution, where radius r(x) the perpendicular distance from the x-axis is:

x	0	1	2	3	4	5	6
r(x)	6.2	5.8	4.0	4.6	5.0	7.6	8.2

Using Simpson's rule with n = 3 and h = 1 **(4 marks)**