TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

EMG 2414: NUMERICAL METHODS FOR ENGINEERS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: JUNE/JULY 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

$$A = \begin{bmatrix} 3 & 2-i \\ 4+3i & -5+2i \end{bmatrix}$$

a) Find the adjoin of a matrix A given **(2 marks)**

b) Using Romberg's' integration method, find the value of $\int_{1}^{1.8} y(x)dx$ starting with trapezoidal rule for the given tabular values below **(6 marks)**

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.73.107	1.8
y =f(x)	1.543	1.669	1.811	1.971	2.151	2.352	2.577	2.828	3.107

c) Verify that L_1, L_2 $ft = L_2 L_1 f(t) = Lf(t)$ given that $L_1 = 2D + 3$ and $L_2 = D^2 + 2D + 1$ while $f(t) = t^3$ **(4 marks)**

d) Consider the matrix $A = \begin{bmatrix} \lambda & 2 \\ 2 & \tau - 3 \end{bmatrix}$ find all possible value of λ to make A non-singular **(3 marks)**

e) Determine the characteristic equation of A, the Eigen values and corresponding Eigen vectors hence general solution for the system x' = Ax given by $A = \begin{pmatrix} 4 & -1 \\ -4 & 4 \end{pmatrix}$ **(8 marks)**

f) Find the Fourier transform of $f(t) = e^{-|t|}$ where $f(t) = \begin{cases} e^{t}, & t < 0 \\ e^{-t} & t \geq 0 \end{cases}$ **(7 marks)**

Question Two

a) Define linear independence of function **(2 marks)**

b) By use of row reduction find the inverse of the matrix A given by $A = \begin{pmatrix} 1 & -1 & -1 \\ 3 & -1 & 2 \\ 2 & 2 & 3 \end{pmatrix}$ **(8 marks)**

c) Solve the following differential equation $\frac{dy}{dt} = t + y$ with the initial condition y(0) = 1, using the fourth order Runge Kutta method from t = 0 to t 0.45 taking h = 0.1 and $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$ where

$$k_1 = nf(tn, yn)$$
$$k_2 = nf\left(t_n + h/2, yn + k_1/2\right)$$
$$k_3 = hf\left(t_n + \frac{h}{2}, y_n + k_2/2\right)$$
$$k_4 = hf(t_n + h, y_n + k_3)$$

(10 marks)

Question Three

a) Determine the adjoint of a matrix A if $A = \begin{pmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{pmatrix}$ hence compute A^{-1} the inverse of the matrix **(8 marks)**

$$\frac{dx}{dt} = 6x - 3y$$
$$\frac{dy}{dt} = 2x + y$$

b) Solve the system **(6 marks)**

c) Use Simpson's one third rule and the composite trapezoidal rule to evaluate the approximate value of $\int_0^1 \frac{1}{1+x^2}\,dx$ taking h = 0.25 (working to 4 dp) compute the solution **(6 marks)**

Question Four

a) Using the Taylor's series for y(x) find y(0.1) if y(x) satisfies $y^1 = x - y^2$ and $y(0) = 1$ **(6 marks)**

b) Solve the simultaneous equation using Crammers rule:

$$x + y + z = 4$$
$$2x - 3y + 4z = 33$$
$$3x - 2y - 2z = 2$$

(7 marks)

c) Find the determinant of matrix A given that $A = \begin{bmatrix} 1+j & 2j \\ -3j & 1-4j \end{bmatrix}$ **(2 marks)**

d) Determine Fourier transform of $f(t) = e^{kt} \quad 0 \le t < \infty$ **(5 marks)**

Question Five

a) Use the Gauss Legendre quadrature formula to compute the integral:

$$I = \int_5^{12} \frac{dx}{x}$$

for n = 3 in the interval (-1, 1) **(5 marks)**

b) Using the D operator method find the value of x from the system:

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$
$$2\frac{dx}{dt} + 2d\frac{y}{dt} + 3x + 8y = 2$$

(7 marks)

c) Convert the equation $\frac{d^4x}{dt^4} + 5\frac{d^3x}{dt^3} + \frac{3d^2x}{dt^2} - \frac{2dx}{dt} + 6x = t^2$ to a normal linear system of four differential equations in four unknowns **(4 marks)**

d) Use mid-ordinate rule with n = 10 to approximate the integral $\int_0^1 e^x\,dx$ **(4 marks)**