TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE
BACHELOR OF SCIENCE IN CIVIL/ELECTRICAL & ELECTRONIC ENGINEERING

SMA 2471: NUMERICAL ANALYSIS

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Use the Euler's method to find an approximation to the initial value problem $\frac{dy}{dx} = y - \frac{2x}{y}$ if y(0) = 1 in the range $0 \le x \le 0.2$ with step size h = 0.1 **(4 marks)**

b) An alternating current i has the following values at equal intervals of 2 milliseconds:

Time (s)	0	2	4	6	8	10	12
Current (A)	0	3.5	8.2	10.0	7.3	2.0	0

Charge q in millicoulumbs is given by $q = \int_0^{12} idt$, use Simpson's rule to determine the approximate charge in the 12ms period. **(3 marks)**

c) If $\frac{dy}{dx} = y = x$ where $y(0) = 2$ find y (0.1) with h = 0.1 using the 4th order Runge –Kutta method correct to 4 d.p. **(7 marks)**

d) Find the unique quadratic polynomial of degree two (2) or less such that $f(0) = 1$ $f(1) = 3$, f(3) = 55 using the Lagrcange interpolation. **(6 marks)**

e) Use the 2 point Gauss-legendre rule to approximate $\int_{-1}^{1} \frac{dx}{x+2}$ **(4 marks)**

f) The table below provides the relationship between length L(m) and temperature T(k) on a structure, find the length L at T = 372.1K using Newton divided difference. **(6 marks)**

T(Kelvin)	361	367	378	387	399
L (metres)	154.9	167.0	191.0	212.5	244.2

SECTION B (Answer any TWO questions from this section)

Question Two

a) Use the trapezoidal rule with 4 intervals to evaluate the integral $\int_1^3 \frac{2}{\sqrt{x}} dx$ correct to 3 d.p. **(4 marks)**

b) Given the first order differential equation $\frac{dy}{dx} = x - y$ subject to the condition that y(0) = 1 and h = 0.1, solve the differential equation using milne's method if $0 \le x \le 4$ correct to 4 significant figures. **(8 marks)**

c) Applying the Newton-Raphson's method determine the root of an equation given by $f(x) = \cos x - xe^x$ correct to 3 d.p. **(8 marks)**

Question Three

a) From the table given below:

x^o	30	60	90
$\cos x^o$	0.866	0.500	0.000

Find $\cos 50^o$ using Newton forward difference interpolating quadratic polynomial **(4 marks)**

b) Solve by Taylor's series the differential equation $xy' = x - y$ if $y(2) = 2$ at x = 2.1 correct to 4 d.p **(8 marks)**

c) Using the error bound determine a value of h to estimate $\int_0^1 e^{-x^2} dx$ correct to 2 decimal places by the trapezoidal rule. **(8 marks)**

Question Four

a) Using the central difference obtain a numerical approximation for the 2nd derivative of $\log_{10} x$ at x = 5 given h = 0.125 **(4 marks)**

b) Determine the value of y when x = 0.1 using Euler's modified method given that $y(0) = 1$ if $\frac{dy}{dx} = y + x^2$ and h = 0.05 **(6 marks)**

c) Use Simpson's rule to evaluate $\int_0^{\frac{\pi}{3}} \sqrt{1 - \frac{1}{3}\sin^2\theta}$ using 6 intervals **(5 marks)**

d) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ with a step size h = 0.25 using the trapezoidal rule. **(5 marks)**

Question Five

a) Obtain the truncation error bound of sin 0.15 when determined by Lagrange linear interpolation if provided with sin 0.1 = 0.0998 and sin 0.2 = 0.1987 **(5 marks)**

b) A particle moves along a path such that at a time t its distance S from a fixed point on the path is given by $\frac{ds}{dt} = t(8 - t^3)^{1/2}$ using the Simpson's $1/3$ rule to calculate the approximate distance travelled by the particle from time t = 0.8 sec to t = 1.6 sec using n = 8 correct to 3 d.p. **(5 marks)**

c) Use the trapezoidal rule to evaluate $\int_0^{\pi/2} \sin x dx$ given that n = 10. **(6 marks)**

d) Illustrating by finite difference tables, explain the phrases central difference and backward difference as used in numerical analysis and state when they are best applied. **(4 marks)**