TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

BACHELOR OF SCIENCE IN MATHEMATICS & COMPUTER SCIENCE

AMA 4213: NUMBER THEORY

END OF SEMESTER EXAMINATION
SERIES: APRIL 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) (i) Use axioms of integers to prove:

$$a \bullet (b + c) = a \bullet b + a \bullet c$$

(2 marks)

$$\prod_{j=1}^{5} j^2$$

(ii) Evaluate **(2 marks)**

(iii) Suppose a, b and c are integers with a < b and c>0 show that a c < b c. **(3 marks)**

b) (i) If a, b, m and n are integers, where c/a and c/b show that c/(ma +nb) **(2 marks)**

(ii) Evaluate G.C.D (15, 21, 35) **(2 marks)**

c) By use of Euclidean Algorithm Evaluate (34, 55) **(3 marks)**

d) Show that there are infinitely many primes of the form 4n + 3, where n is a positive integer. **(3 marks)**

e) By use of Wilson's theorem, show that 7 is prime. **(4 marks)**

f) Let d and n be positive integers such that d divides n, show that 2^d-1 divides 2^n-1. **(3 marks)**

g) Let "p" be a prime number and "a" a positive integer show that $\phi(p^a) = p^a - p^{a-1}$ **(3 marks)**

h) Let a = 1,028 and b = 34. Find the values of q and r by use of division algorithm. **(3 marks)**

Question Two

a) Let a, b and c be integers with (a, b) = d. Show that:

(i) $\left(a/d, b/d \right) = 1$ **(4 marks)**

(ii) $(a + cb,\ b) = (a, b)$ **(4 marks)**

b) By use of division algorithm, find the base 2 expansion for 1864. **(5 marks)**

c) Show that the greatest common divisor of the integers a and b that are not both zero is the least positive integer that is a linear combination of a and b. **(7 marks)**

Question Three

a) Factor 6077 using the method of format factorization. **(7 marks)**

b) Without performing the division show that the Format's number:

$F_5 = 2^{2^5}$ is divisible by 641. **(6 marks)**

c) Given a linear Diophantine equation as $20x + 50y = 510$ find all its solutions. **(7 marks)**

Question Four

a) If a and b are integers, show that $a \equiv b$ (mod.m) if and only if there is an integer k such that a = b +km. **(7 marks)**

b) If a, b c and m are integers with m>0 such that $a \equiv b$ (mod.m) show that:

(i) $a + c \equiv b + c$ (mod.m) **(2 marks)**

(ii) $a - c \equiv b - c$ (mod.m) **(2 marks)**

(iii) $ac \equiv bc$ (mod.m) **(3 marks)**

c) Show that each of the following congruences hold:

(i) $22 \equiv 7$ (mod.5) **(2 marks)**

(ii) $-3 \equiv 30$ (mod.11) **(2 marks)**

(iii) $666 \equiv 0$ (mod.37) **(2 marks)**

Question Five

a) Let a and b be non-negative integers show that 22 $(a,b)\bullet(a,b) = ab$ **(6 marks)**

b) If x and y belong to the same residue class modulo m, show that (x, m) = (y, m) **(3 marks)**

c) Solve the congruence $296x \equiv 176$ (mod.114) **(7 marks)**

d) Consider the congruence:

22 $x^{11} + 2x^8 + x^5 + 3x^4 + 4x^3 + 1 = 0$ (mod.5) divide this congruence by $x^5 - x$ **(4 marks)**