TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR:

SMA 2471: NUMERICAL ANALYSIS I

END OF SEMESTER EXAMINATION
SERIES: APRIL 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **TWO** printed pages

Question One (Compulsory)

a) Find a polynomial that will fit (0, -4), (1, -3) and (3, 5). Hence find the rate of change of y with respect to x. When x = 1.5. **(4 marks)**

b) Use Newton-Raphson method to evaluate $10^{2/3}$ to 7 decimal places taking $x_0 = 4$ as the initial approximation. **(6 marks)**

c) Solve $y' = y,\ y(0) = 1$ at x = 0.04 using Taylor series. **(5 marks)**

d) A coach accelerates from rest to 100km/h in 90 seconds. Its speed, vkm/h measured at five second intervals, is given by the table:

t	0	5	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9

			0	5	0	5	0	5	0	5	0	5	0	5	0	5	0	5	0
v	0	5	19	22	32	40	43	52	61	65	66	71	77	85	90	95	98	99	100

Use Simpson's rule to find the total distance travelled within the 90 seconds. **(4 marks)**

$$I = \int_1^2 \frac{2x}{1+x^4} dx$$

e) Evaluate the integral using Gauss Legendre 3 point formula. **(6 marks)**

λ_1 λ_2 $\int_0^h f(x)dx = \lambda_1 f(0) + \lambda_2 f(h) + E$

f) Determine and so that is exact and write down the integration formula (NB E is the error term) **(5 marks)**

Question Two

$\int_0^2 e^x dx$

a) Use Romberg integration to evaluate correct to 3 decimal places. **(12 marks)**

$\int_0^2 e^{x} dx$

b) How many subdivisions are required to approximate the integral with trapezoidal rule and with the accuracy in (a) above? **(8 marks)**

Question Three

a) The response x of a given hydraulic value subject to a sinusoidal input variation is given by $\frac{dx}{dt} = \sqrt{2\left(1 - \frac{x^2}{\sin^2 t}\right)}$ $\left(\frac{dx}{dt}\right)_{x=0} = \sqrt{\frac{2}{3}}$ with x = 0 at t = 0. Show that and hence use Runge-Kutta fourth order method to obtain a solution at t = 0.2 **(10 marks)**

$$\int_{x0}^{x1} f(x)dx = \frac{h}{2}(f_0 + f_1) + ph^2(f'_0 - f'_1) + R$$

b) Obtain a generalized trapezoidal rule of the form find the constant P and the error term. **(10 marks)**

Question Four

a) Use Newton-Gregory forward difference formula to establish the integration formula $\int_{x_0}^{x_n} y dx = nh\left\{y_0 + \frac{n}{2}\Delta y_0 + n\left(\frac{2n-3}{12}\right)\Delta^2 y_0 + .\frac{n(n-2)}{24}\Delta^3 y_0 + ...\right\}$

(4 marks)

$$y_4 = y_0 + \frac{4}{3}h(2y_1 - y_2 + 2y_3)$$

b) By putting n = 4 show that **(3 marks)**

c) Solve the differential equation: $\frac{dy}{dx} = x - y$ by Milne's method to 4 decimal places given that the modified Euler method gives y_1, as 0.9095 and y_2 as 0.8372 . **(13 marks)**

Question Five

a) Find a polynomial to fit: **(11 marks)**

x	0	0.1	0.2	0.3	0.4	0.5
Y	1	0.521	-0.112	-0.893	-1.816	-2.875

b) Compute by Gaussian quadrature:

$$I = \int_0^1 \frac{\ln|x+1|}{\sqrt{x(1-x)}}\, dx$$

The error must not exceed 5×10^{-3} **(9 marks)**