TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF TECHNOLOGY IN APPLIED PHYSICS
BACHELOR OF TECHNOLOGY IN RENEWABLE ENERGIES
(BTAP/BTRE)

AMA 4216: LINEAR ALGEBRA

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define what is meant by:

(i) Diagonal matrix **(1 mark)**

(ii) An upper triangular matrix **(1 mark)**

b) For what values of x, y and z is the matrix:

$$A = \begin{pmatrix} x & y & z \\ 2 & 0 & 3 \\ 4 & 3 & 3 \end{pmatrix}$$

is symmetric **(2 marks)**

$$\underset{\sim}{a} = 6\underset{\sim}{i} + 8\underset{\sim}{j} - 4\underset{\sim}{k} \qquad \underset{\sim}{b} = m\underset{\sim}{i} + 5\underset{\sim}{j} - 3\underset{\sim}{k}$$

c) Determine the value of M so that and are orthogonal. **(2 marks)**

d) Find the value of x if the given matrix A is singular:

$$A = \begin{pmatrix} 2 & 2x & 2x^2 \\ 2 & 2 & 2 \\ 1 & -6 & 18 \end{pmatrix}$$

(3 marks)

e) Find the equation of the plane through the point (-2, 4, 6) and perpendicular to the plane $4x - 6y + 8z = 2$ and $6x - 10y + 4z = 6$ **(4 marks)**

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$

f) Let find all Eigen values of A and the corresponding Eigen vector. **(5 marks)**

g) Solve using Cramer's rule **(6 marks)**

$$x + y - z = 1$$
$$x - y + 2z = 3$$
$$2x - y + z = 5$$

h) For which value of m will the vector $u = (1,-2,m)$ in $\Re^3$ be a linear combination of $v = (3,0,-2)$ and $w = (2,-1,-5)$

(6 marks)

Question Two

$$A = \begin{pmatrix} -1 & 2 & -3 \\ 2 & 1 & 0 \\ 4 & -2 & 5 \end{pmatrix}$$

a) Find the inverse of the matrix **(5 marks)**

$$\begin{pmatrix} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{pmatrix}$$

b) Find the row reduced echelon form of the matrix: **(5 marks)**

c) Solve by determinants

$$-4y + x = 6 - z$$
$$4x + 2z = -1 + y$$
$$-3z + 20 = -2x - 2y$$

(6 marks)

d) Find the value of K so that the plane $(k+1)x - y + (2-k)z = 0$ is perpendicular to the plane $2x + 6y - z + 3 = 0$

(3 marks)

Question Three

a) Find the dimension and a basis of the solution space w of the system:

$$x + 2y + 2z - s + 3s = 0$$
$$x + 2y + 3z + s + t = 0$$
$$3x + 6y + 8z + s + 5t = 0$$

(8 marks)

b) Find all Eigen values and basis for each Eigen space:

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

(9 marks)

c) Obtain the acute angle between the two planes $x + 2y - 2z = 0$ and $3x - 5y + 4z = 0$ **(3 marks)**

Question Four

a) Find the area of a triangle with vertices at (4, -3, 1), (3, -1, 2) and (1, -1, -3) using vector approach.

(4 marks)

b) Find the unit vector perpendicular to the vectors $\underset{\sim}{A} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{B} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ **(3 marks)**

c) Solve by use of inverse matrix method:

$$x_1 + 3x_2 + 2x_3 = 3$$
$$2x_1 + 4x_2 + 2x_3 = 8$$
$$x_1 + 2x_2 - x_3 = 10$$

(6 marks)

d) Obtain the distance from the point (2, -3, -1) to the plane $2x - 3y + 6z + 7 = 0$ **(3 marks)**

e) Find the equation of the plane through the point (4, 3, 6) and perpendicular to the line joining that point to the point (2, 3, 1) **(4 marks)**

Question Five

a) Write $v = t^2 + 4t - 3$ as a linear combination of $e_1 = t^2 - 2t + 5 \quad e^2 = 2t^2 - 3t, \; e^3 = t + 3$

(8 marks)

b) Find the equations of the plane passing through the point (-1, 2, 4) and containing the line of intersection of planes $5x - y + z = 1$ and $x - 6y + z = 2$ **(7 marks)**

c) Find the dimension and a basis for one vector space spanned by (1, 4, 3), (2, -2, 6) and 91, -6, 3) **(5 marks)**