TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN MARINE RESOURCE MANAGEMENT

SMA 2279: LINEAR & BOOLEAN ALGEBRA

END OF SEMESTER EXAMINATION
SERIES: APRIL 2015
TIME ALLOWED: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FIVE** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Show that $AB^{-1} = B^{-1}A^{-1}$ **(3 marks)**

b) Find the area of a parallelogram with vertices at (1, 1, 1) (2, 3, 2) (-2, 4, 4) and (-3, 2, 3) **(5 marks)**

c) Show that the lines $L_1 : \vec{r}_1 = (4,1,2) + t(1,2,-1)$ and $L_2 : \vec{r}_2 = (-1,5,-1) + t3(-1,1)$ are perpendicular **(3 marks)**

d) Determine the unknown quantities in the following expression:

$$2\begin{pmatrix} x+2 & y+3 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ y & z \end{pmatrix}^T$$

(5 marks)

e) Find the characteristic polynomial of the matrix $A=\begin{pmatrix}-2 & -1\\ -1 & 2\end{pmatrix}$ and hence show that A satisfies its own characteristic equation **(5 marks)**

f) Convert (58.32) to binary **(4 marks)**

g) Construct the truth tables of $\sim(p\wedge q)$ and $\sim p\vee\sim q$ and hence make a conclusion **(5 marks)**

Question Two

a) Resolution of forces and balancing of moments leads to the following equation for three forces F1, F2, F3 (Newtons) acting on one of the struts in an aircraft wing;

$$F_1 - F_2 = 0$$
$$2F_1 + F_2 - 2F_3 = 20$$
$$F_2 - F_3 = 4$$

Find the forces by Crammer's rule **(7 marks)**

b) Find all the eigen values and eigen vectors of the followign matrix **(8 marks0**

$$A=\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$$

c) Show that for any vectors $\vec{a},\vec{b},\vec{c}$ in $\Re^3$ we have

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}(c\times a)+c\times(a\times b)=0$$

(5 marks)

Question Three

a) Reduce the following matrix to echelon form and state the rank of the matrix. **(6 marks)**

$$A=\begin{pmatrix}1 & 2 & -3 & 0\\ 2 & 4 & -2 & 2\\ 3 & 6 & -4 & 3\end{pmatrix}$$

as its general solution **(5 marks)**

b) Apply the Gauss-Jordan method to solve the following system of equations **(9 marks)**

$$4y+z=2$$
$$2x+6y-2z=3$$
$$4x+8y-5z=4$$

Question Four

a) Find the determinant of the following matrix **(6 marks)**

$$A = \begin{pmatrix} 1 & -2 & 3 & 1 & 2 \\ 1 & 1 & 4 & -1 & 3 \\ 2 & 5 & 9 & -2 & 8 \end{pmatrix}$$

b) Reduce to row canonical form **(8 marks)**

c) Define the following terms:

(i) Proposition

(ii) Tautology

(iii) Contradition **(6 marks)**

Question Five

a) Solve the following homogeneous system of equations:

$$x_1 + 2x_2 \qquad x_4 = 0$$

$$-2x_1 - 3x_2 \quad + 4x + 5x_4 = 0$$

$$2x_1 + 4x_2 \quad - 2x_4 = 0$$

(7 marks)

b) Attempt to solve the following system using Gaussian elimination and explain what occur to indicate that the system is impossible to solve:

$$-x_1 + 3x_2 - 2x_3 = 1$$

$$-x_1 + 4x_2 - 3x_3 = 0$$

$$-x_1 + 5x_2 - 4x_3 = 0$$

(7 marks)

$$(p \to q)(q \to r)(p \to r)$$

c) Find the truth table for the statement **(6 marks)**