TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

SMA 2279: LINEAR & BOOLEAN ALGEBRA

SPECIAL/SUPPLEMANTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

Instructions to Candidates:
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Define the following terms:
- **(i)** Orthogonal vectors **(1 marks)**
- **(ii)** A diagonal matrix **(1 marks)**
- **(iii)** A compound proposition **(1 marks)**

b) Use your knowledge of the truth tables to determine the truth values of the following compound statements.

2 + 3 = 5 and 1 + 1 =3 **(3 marks)**

c) Find the unit vectors that are tangent and normal to the following curve at the stated point.

$$3x^2 + 8xy + 2y^2 - 3 = 0$$

at (1, 0) **(5 marks)**

d) If $\underset{\sim}{a} = 4\underset{\sim}{i} + 3\underset{\sim}{k}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + j5k$ find:

(i) $\underset{\sim}{a} + \underset{\sim}{b}$ **(2 marks)**

(ii) $\left|2\underset{\sim}{a} + 3\underset{\sim}{b}\right|$ **(3 marks)**

e) If $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 0 & 4 \\ 6 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 9 & 8 \\ 2 & 3 & 6 \\ 1 & 5 & 0 \end{bmatrix}$

Evaluate:

(i) $A + B$ **(2 marks)**

(ii) AB **(3 marks)**

f) Construct a truth table for the statement $\sim p \wedge q$ **(4 marks)**

g) Find the inverse of the matrix $\begin{bmatrix} 2 & -3 \\ 4 & 2 \end{bmatrix}$ hence solve the simultaneous equation:

$2x - 3y = -4$

$4x + 2y = 8$

(5 marks)

Question Two

a) Find an equation for the plane through the points A(0,0,1) B(2,0,0) and C(0,3,0) **(6 marks)**

b) Find a unit vector perpendicular to both $A = 2i + j - k$ and $B = i - j + 2k$ **(4 marks)**

c) Find the area of the triangle whose vertices are $A(1,-1,0)$ $B(2,1,-1)$ and $C(-1,1,2)$ **(5 marks)**

d) Find the distance from the point S(1, 1, 5) to the line L; $x = 1 + t,\ y = 3 - t;\ 2 = 2t$ **(5 marks)**

Question Three

a) Define the following terms:

(i) Logic **(1 mark)**

(ii) Tautology **(1 mark)**

b) Show that A – B = $A \cap B'$ where A and B are subjects of the universal μ **(5 marks)**

c) Construct a truth table for the statement $p \rightarrow (q \wedge \sim r)$ **(5 marks)**

d) Show that the conditional statement $[(p \wedge q) \wedge p] \rightarrow q$ is a tautology **(4 marks)**

e) Determine the values of x that make the following statements to be true.

$4 + 3 = 5$ if and only if $x + 3 = 9$ **(3 marks)**

Question Four

a) Define the following terms:

(i) Null matrix **(1 mark)**

(ii) Triangular matrix **(2 marks)**

(iii) Principal minors **(2 marks)**

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

b) If find the inverse of A **(6 marks)**

c) Use Cramer's rule to solve.

$x + 2y + 3z = 6$

$2x + y + z = 5$

$3x + y - 2z = 1$

(7 marks)

d) Two factory outlets F1 and F2 in New York and Los Angeles sell sofas (s), chairs © and tables. T with a profit of 110, 45 and 80 respectively. The sales in a certain week were given by the matrix.

$$A = \begin{array}{c} \begin{matrix} S & C & T \end{matrix} \\ \begin{bmatrix} 600 & 400 & 100 \\ 300 & 820 & 205 \end{bmatrix} \begin{matrix} F1 \\ F2 \end{matrix} \end{array}$$

Find the total profits for F1 and F2 **(2 marks)**

Question Five

a) Define the terms Eigen value and Eigen vector **(2 marks)**

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

b) If calculate:

(i) Eigen values of A **(3 marks)**

(ii) Eigen vectors of A **(4 marks)**

c) Find a Cartesian equation for the plane through $P0(-3,0,7)$ perpendicular to the vector $wv = 5i + 2j - k$

(3 marks)

d) If $\vec{A} = 2i - 3j + 7k$, determine the direction of $\vec{A}$ **(3 marks)**

e) Show that the conditional and contra positive statements are equivalent and that the converse and inverse statement are equivalent. **(5 marks)**