



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Engineering and Technology

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

DIPLOMA IN TECHNOLOGY
ELECTRICAL POWER ENGINEERING
MECHATRONICS
TELECOMMUNICATION & INFORMATION ENG.

ENGINEERING MATHEMATICS VI

SEMESTER VI FINAL EXAMINATIONS

SERIES: AUGUST/SEPTEMBER 2011

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- *Question paper*
- *Answer booklet*
- *A Non-programmable calculator*
- *SMP Table*
- *Abridged Laplace Transforms table*

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

Find an attached copy of Abridged Laplace Transform Table

This paper consists of **THREE** printed pages

Question 1 (Compulsory)

- a) Given $\begin{pmatrix} 3 & 1 \\ 4 & 3 \end{pmatrix}$ and the characteristic polynomial $f(\lambda) = \det(A - \lambda I)$, solve the characteristic equation $f(\lambda) = 0$ and hence show that $f(A) = 0$ where $0 = \text{Zero matrix}$. (7 marks)

- b) Determine the eigenvalues of the following matrix.

$$A = \begin{pmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{pmatrix} \quad (5 \text{ marks})$$

- c) (i) Determine the Laplace transform of $t^2 e^{-2t} \cos t$ (7 marks)

$$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$

- (ii) Given the function

- i) Express the function in terms of unit step (Heaviside) function
 ii) Determine its Laplace transform (4 marks)

- d) (i) Show that the function $e^x(\cos y + j \sin y)$ is analytic function.
 (ii) Determine the derivative of the function in d(i) (7 marks)

Question 2

- a) Given the matrix

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

Determine

- i) Eigen value of A
 ii) Eigenvectors of A
 iii) Diagonalize the matrix A such that $P^{-1}AP = S$ where S is a diagonal matrix (14 marks)

$$C = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

- b) Determine the eigenvectors of the matrix given (6 marks)

Question 3

$$x^2 - y^2 + 2y$$

- a) Given the function
- i) Show that the given function is harmonic (3 marks)
- ii) Show that the function remains harmonic under the transformation $Z = W^3$ (5marks)

- b) Given $U - V = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + jv$ is an analytic function of $z = x + jy$
determine $f(z)$ in terms of z (2 marks)

Question 4

$$\frac{1 - \cos t}{t}$$

- a) Determine the Laplace transform of (6 marks)
- b) Given the function

$$f(t) = \begin{cases} \sin 2t, & 2\pi < t < 4\pi \\ 0, & \text{otherwise} \end{cases}$$

- i) Represent $f(t)$ in terms of unit function
- ii) Hence, determine its Laplace transform (6 marks)

$$f(t) = \begin{cases} \sin wt & \text{for } 0 < t < \frac{\pi}{w} \\ 0 & \text{for } \frac{\pi}{w} < t < \frac{2\pi}{w} \end{cases}$$

- c) The Half wave rectifier function is given by $f(t)$
determine the Laplace transform of

Question 5

$$w = \frac{1}{z},$$

$$x^2 - y^2 = 1$$

- a) Show that under the transformation the image of the hyperbola is the lemniscate $R^2 = \cos 2\phi$ (6 marks)
- b) The vertices of a triangle in the z - plane is given by i , $1+i$ and $1-i$

- i) Determine its image under the transformation $w = e^{5\pi i/3} \cdot z - 2 + 4i$ (6 marks)
- ii) On a graph paper plot the triangle on the z-plane and its image on the w-plane. (5 marks)

- c) Transform the curve $x^2 - y^2 = 4$ under the mapping $w = z^2$ (3 marks)