# TECHNICAL UNIVERSITY OF MOMBASA

# Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

**UNIVERSITY EXAMINATION FOR DEGREE IN BACHELOR OF SC. IN MECHANICAL & AUTOMOTIVE ENGINEERING**

EMG 2414: NUMERICAL METHODS FOR ENGINEERS

**SPECIAL/SUPPLEMENTARY EXAMINATION**
**SERIES:** OCTOBER 2013
**TIME:** 2 HOURS

**Instructions to Candidates:**
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

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**SECTION A (COMPULSORY)**

**Question One (30 marks)**

**a)** An alternating current I has the following values at equal intervals of 2 milliseconds:
b)

| **Time (ms)** | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| **Current (A)** | 0 | 3.5 | 8.2 | 10.0 | 7.3 | 2.0 | 0 |

$$q = \int_0^{12} i\,dt$$

Charge q in millicoulombs is given by use Simpson's rule to determine the approximate charge in 12ms period. **(3 marks)**

**c)** A dc circuit comprises three closed loops, applying Kirchhoff's laws the closed loops gives the following equations for current flow in milliampres:

$$2I_1 + 3I_2 - 4I_3 = 26$$
$$I_1 - 5I_2 - 3I_3 = -87$$
$$-7I + 2I_2 + 6I_3 = 12$$

Use the Cramer's rule to determine the currents $I_1, I_2$ and $I_3$. **(6 marks)**

**d)** Solve simultaneously the system

$$\frac{dx}{dt} = 6x - 3y$$
$$\frac{dy}{dt} = 2x + 2y$$

**(6 marks)**

**e)** If $\frac{dy}{dx} = y - x$ where $y(o) = 2$ find y(0.1) with h = 0.1 using the 4th order Runge-Kutta method correct to 4 d.p **(7 marks)**

**f)** Determine whether the solutions of the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ are linearly independent in the internal $0 < x < \infty$ **(5 marks)**

**g)** Find $L_1L_2f(t)$ given that $L_1 = 2D + 3$ $L_2 = D^2 + 2D + 1$ and $f(t) = t^3$ **(3 marks)**

**SECTION B (Answer any TWO questions from this section)**

**Question Two (20 marks)**

**a)** Evaluate $\int_0^1 \frac{1}{1+x}dx$ correct to 4 significant figures using Gauss-Legendre 3- point formula on an interval (-1, 1) **(6 marks)**

**b)** Given the first order differential equation $\frac{dy}{dx} = x - y$ subject to the condition y(o) = 1, h = 0.1, solve the differential using modified Euler method if $0 \le x \le 0.3$ correct to 4 significant figures. **(8 marks)**

**c)** Using trapezoidal rule evaluate $\int_0^{\pi/2} \sin x dx$ given that the interval is divided into 10 equal parts. **(6 marks)**

**Question Three (20 marks)**

**a)** Use Romberg method to computer the $\int_0^1 \frac{dx}{1+x}$ correct to 3 d.p given that for h = 0.5, $I(h) = 0.7084,\ I\left(\frac{h}{z}\right) = 0.6970$ and $I\left(\frac{h}{4}\right) = 0.6941$ **(5 marks)**

**b)** By Taylor's series for $y(x),$ find y(0.1) correct to 4 d.p if $y(x)$ satisfies $y' = x - y^2$ and $y(o) = 1$ **(7 marks)**

**c)** Using the operator method to solve simultaneously the system

$$2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = t$$

$$2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 2$$

**(8 marks)**

**Question Four (20 marks)**

**a)** If $\frac{dy}{dx} = y - x$ where $y(0) = 2$ find y(0.1) then y(0.2) with h(0.1) using the 2nd order Runge-Kutta method correct to 4d.p **(5 marks)**

**b)** Apply the gauss quadrature formula to compute the integral $I = \int_5^{12} \frac{dx}{x}$ choosing n = 3 over the interval (-1,1) **(6 marks)**

**c)** Using matrix method solve the system:

$$\frac{dx_1}{dt} = 3x_1 + x_2 - x_3$$

$$\frac{dx_2}{dt} = x_1 + 3x_2 - x_3$$

$$\frac{dx3}{dt} = 3x_1 + 3x_2 - x_3$$

**( 9 marks)**

**Question Five (20 marks)**

a) Use the Euler's method to find an approximation to the initial value problem $\frac{dy}{dx} = y - \frac{2x}{y}$ if y(0) = 1 in the range $0 \le x \le 0.2$ with the step size h = 0.1 **(6 marks)**

b) A particle moves along a path such that at a time t its distance S from a fixed point on the path is given by $\frac{ds}{dt} = t\left(8 - t^3\right)^{1/2}$

Use Simpson's $\frac{1}{3}$ rule to calculate the approximate distance travelled by the particle from time $t = 0.8\sec$ to $t = 1.6\sec$ using 8 stips. (working correct to 3 d.p) **(6 marks)**

c) Evaluate the determinant of $\begin{bmatrix} i^2 & (1+i) & 3 \\ (1-i) & 4 & i \\ 0 & i & 5 \end{bmatrix}$ using the 1st column where I is a complex number. **(4 marks)**

d) Use the 2 point, Gause-legendre rule to approximate $\int_{-1}^{1} \frac{dx}{x+2}$ **(4 marks)**