THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF JKUAT)

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL AND AUTOMOTIVE ENGINEERING

DIPLOMA IN MECHANICAL ENGINEERING

DIPLOMA IN CHEMICAL ENGINEERING (PRODUCTION)

DIPLOMA IN MECHANICAL (AUTOMOTIVE ENGINEERING)

AMA 2204

ENGINEERING MATHEMATICS

YEAR 2 SEMESTER II EXAMINATION

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: MARCH, 2012

TIME: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. You should have the following for this examination:
   - Answer Booklet
   - Mathematical table/Scientific calculator
   - Drawing Instruments
   - Abridged Laplace transforms Table

2. Answer questions **ANY THREE questions** from the following **FIVE .**

3. Maximum marks for each part of the question are as shown.

4. This paper consists of ***FOUR printed pages.***

QUESTION ONE

a) i State the necessary and sufficient condition for an equation $Mdx + Ndy = 0$ to be exact

ii Show that equation $\left[(\cos x)\ln(2y-8)+\frac{1}{x}\right] dx + \frac{\sin x}{y-4}dy = 0$.

is exact and solve the differential equation given that Given $y(1) = \frac{9}{2}$ **(7marks)**

b) Show that the Laplace Transform of $2\cos(2t+\beta)$ is $\frac{2}{s^2+4}(s\cos\beta - 2\sin\beta)$ where β is a constant. **(7marks)**

c) Determine the inverse Laplace Transform of $\frac{26-s^2}{s(s^2+4s+13)}$, **(6marks)**

QUESTION TWO

(a) The instantaneous current I passing through a circuit of resistance R and inductance L satisfies the differential equation. $L\frac{di}{dt} + Ri = V_o \cos\omega t$ Where t is time and V_o and ω are constant. Show that $i = \frac{V_o}{\omega^2 L^2 + R^2}\{ L\omega \sin\omega t + R\cos\omega t\} + Ce^{-\frac{R}{L}t}$

using an integrating factor. **(10marks)**

b) Solve the following differential equation completely.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = 3x + 1, \textit{given that } y = 0, \frac{dy}{dt} = 0, \textit{when } t = 0.$$

(10marks)

QUESTION THREE

a) Determine the inverse Laplace transform of

i) $\dfrac{7s^2-6s-64}{(s-2)(s^2-16)}$

ii) $\dfrac{4s^2-5s+6}{s^3+s^2+4s+4}$ **(8marks)**

b) Given that $x_0 = x_1 = 0$ solve using Laplace transform. $\dfrac{d^2y}{dx^2}+\omega^2 x = E_0 Sinnx$
When

(i) $\omega^2 = n^2$

(ii) $\omega^2 \neq n^2$ **(12marks)**

QUESTION FOUR

Solve the following first order differential equations

I) $\tan x\dfrac{dy}{dx}+y = x^2\tan x$ **(7marks)**

III) $x\,y\,\dfrac{dy}{dx} = x^2+y^2$ *given that* $x=1, y=4$ **(4marks)**

Solve the differential equation $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+2y = 5\sin x,$ *given that* $y(0)=0,\ y'(0)=5$ using

D-operator **(9marks)**

QUESTION FIVE

a) Solve the differential equation completely by using D-operator method

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y = x^2+e^{2x}, \textit{given that } x=0, y=-1, y'=0$$

(13marks)

b) Determine the laplace transform of $t\sin t$ from first principles **(7marks)**

TABLE OF SOME LAPLACE TRANSFORMS

	Function.	***Transform***
	$F(t)$	$\int_0^\infty e^{-st} f(t)\,dt$
1.	1	$1/s$
2.	e^{at}	$1/s-a$
3.	$\sin at$	a/s^2+a^2
4.	$\cos at$	s/s^2+a^2
5.	t	$1/s^2$
6.	$t^n(n\ \ a\ \ positive\ \text{int}\,eger)$	$n!/s^{n+1}$
7.	$\sinh at$	a/s^2-a^2
8.	$\cosh at$	s/s^2-a^2
9.	$t\sin at$	$2as/(s^2+a^2)^2$
9(a)	$t\cos at$	$s^2-a^2/(s^2+a^2)^2$
10.	$\sin at - at\cos at$	$2a^3/(s^2+a^2)^2$
11.	$e^{-at}t^n$	$n!/(s+a)^{n+1}$
12.	$e^{-bt}\cos at$	$(s+b)/\left[(s+b)^2+a^2\right]$
13.	$e^{-bt}\sin at$	$a/\left[(s+b)^2+a^2\right]$
14.	$e^{-bt}\cosh at$	$(s+b)/\left[(s+b)^2-a^2\right]$
15.	$e^{-bt}\sinh at$	$a/\left[(s+b)^2-a^2\right]$
16.	$H(t), H(t-a)$	$1/s, e^{-as}/s$
17.	$\delta(t), \delta(t-a)$	$1,\ e^{-as}$

Some Theorems used in Laplace Transforms

1 If $f(s) = \text{l}\{F(t)\}$ then $f(s+a) = \text{l}\{e^{-at}F(t)\}$

2 $\text{l}\left\{\dfrac{dx}{dt}\right\} = s\,\overline{x} - x_0$

$\text{l}\left\{\dfrac{d^2x}{dt^2}\right\} = s^2\,\overline{x} - s\,x_0 - x_1$

$$\mathcal{L}\left\{\frac{d^3x}{dt^3}\right\} = s^3\,\bar{x} - s^2\,x_0 - sx_1 - x_2$$

and generally

$$\mathcal{L}\left\{\frac{d^nx}{dt^n}\right\} = s^n\,\bar{x} - s^{n-1}\,x_0 - s^{n-2}x_1 \ldots - sx_{n-2} - x_{n-1}$$

3. $$\mathcal{L}\left\{\int_0^t F(t)\,dt\right\} = \frac{1}{s} f(s)$$

4 $$\int_0^t H(X-a)dX = (x-a)H(x-a) \quad , \quad \int_0^t (X-a)^n H(X-a) = \frac{(x-a)^{n+1}}{n+1} H(x-a)$$