THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

UNIVERSITY EXAMINATIONS

EXAMINATION FOR THE DEGREE OF BACHELOR OF ENGINEERING IN ELECTRICAL AND ELECTRONICS/MECHANICAL /CIVIL ENGINEERING

SECOND YEAR SEMESTER I

SCHOOL BASED SUPPLEMENTARY EXAMINATION

SMA 2270: CALCULUS III

DATE: SEPTEMBER 2011 TIME: 2 Hours

INSTRUCTIONS: Answer Question ONE and any other TWO

QUESTION ONE (30 MARKS)

a) Evaluate the following limits

(i) $\lim_{x\to -2} \dfrac{x^3+2x^2-1}{5-3x}$ (4 marks)

(ii) $Lim_{x\to\infty}\, x\sin\dfrac{1}{x}$ (4 marks)

b) State the Rolle's Theorem (4 marks)

c) Verify the validity of Rolle's Theorem for the function $f(x)=x^3-6x^2+11x-6$

(5marks)

d) Find the area of the region enclosed by the parabolas $y=x^2$ and $y=2x-x^2$

(6 marks)

e) Find the value of $\dfrac{df}{dt}$ at $t=\dfrac{\pi}{2}$ if $f(x,y)=xy$ and $x=\cos t$, $y=\sin t$

(7 marks)

QUESTION TWO (20 MARKS)

a) The semicircle $y=\sqrt{r^2-x^2}$ is revolved about the x-axis to generate a sphere. Find the volume of the sphere (6 marks)

b) Find the value of $\dfrac{dy}{dx}$ at $t=-2$ for the function $f(x,y,z)=\sqrt{x^2+y^2+z^2}$

$x=2+t$, $y=-t-1$, $z=t$

(5 marks)

c) Evaluate the following integral

i) $\int_0^2 \int_{x^2}^{2x} (4x+2)\, dydx$ (4 marks)

ii) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dxdydz$ (5 marks)

QUESTION THREE (20 MARKS)

a) State Mean Value Theorem (4 marks)

b) Find the value of $c \in (a,b)$ guaranteed by Mean Value Theorem for $f(x) = x^2 + 2x - 1,\ a = 0,\ b = 1$

(6 marks)

c) Find the Maclaurin series for $f(x) = \ln(x+1)$ up to the term in x^5

(10 marks)

QUESTION FOUR (20 MARKS)

a) The arc of the parabola $y = x^2$ from $(1,1)$ to $(2,4)$ is rotated about the y-axis. Find the area of the resulting surface . (10 marks)

b) Evaluate $\lim_{(x,y)\to(1,1)} \frac{x^2 - 2xy + y^2}{x - y}$, $x \neq y$ (4 marks)

c) Find the Taylor series generated by $f(x) = \cos x$ at $a = 2\pi$ (6 marks)

QUESTION FIVE (20 MARKS)

a) If $z = x + f(u)$ where $u = xy$, show that $x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x$ (5 marks)

b) Find the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $x = 1$ and whose top lies in the plane $z = f(x,y) = 3 - x - y$ (6 marks)

c) Show that the function $f(x) = \begin{cases} \dfrac{xy^2}{x^2+y^4}, & (x,y) \neq 0 \\ 0, & (x,y) = 0 \end{cases}$ is continuous at every point except at the origin. (5 marks)

d) Find $\dfrac{\partial^3 f}{\partial x \partial y \partial z}$ if $f(x,y,z) = xy + yz + zx$ (4 marks)