PP2

TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Science in Electrical Engineering

EEE 4410: Control Engineering III

END OF SEMESTER EXAMINATION

SERIES: NOVEMBER 2024 EXAM

TIME: 2 HOURS

DATE: NOVEMBER 2024

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five Questions; Attempt Question ONE and any other TWO Questions.

Do not write on the question paper.

Question ONE

(a) Define state of a system, state variables, state space and state vector, then state the advantages of state space analysis. (5 marks)

(b) A system is described by the following differential equation. Represent the system in state space.

$\dddot{x}=3\ddot{x}+4\dot{x}+4x=u_1(t)+3u_2(t)+4u_3(t)$ (3 marks)

(c) A feedback system has a closed loop transfer function:

$$\frac{Y(s)}{U(s)}=\frac{10s+40}{s^3+s^2+3s}$$

Construct three different state models showing block diagram in each case. (8 marks)

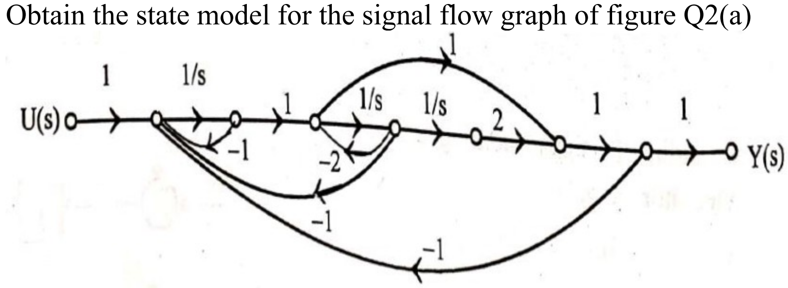
(d) Find the time response of the system described by the equation

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}-1 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u \quad \text{given that} \begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix}=\begin{bmatrix}-1\\ 0\end{bmatrix}, \quad u(t)=t>0$$

(14 marks)

Question TWO

(a) Obtain the state model for the signal flow graph of figure Q2(a)



(6 marks)

Figure Q2 (a).

(b) Compute the state transition matrix by infinite series method given the matrix

$$A=\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

(6 marks)

(c) Determine the Eigen values and Eigen vectors of $A=\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (8 marks)

Question THREE

(a) Explain how poles of a closed loop control system can be placed at the desired points on the s plane (4 marks)

(b) Explain the design of state observer. (4 marks)

(c) Consider the system with state equation $\dot{x}=Ax+Bx$

Where,

$$A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix},\quad B=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By using state feedback control $u=-Kx(t)$, it is desired to have the closed loop poles at $\mu_1=-2+j4$, μ_2-2-j4 and $\mu_3=-10$ .Determine the state feedback matrix K

(12 marks)

Question FOUR

(a) Determine the z- transforms for the following continuous functions:

i) step function $f_1(t)=u(t)$

ii) ramp input $f_2(t)=at$

(8 marks)

(b) A sampled data system has the following transfer function: $G(s)=\dfrac{1}{s+1}$. If the sampling time is 1second and the system is subjected to a unit step input, determine the discrete time response. Note that the system has no zero-order hold included. (8 marks)

(c) Find the transfer function of the following discrete system

$y(k)+2y(k-1)-y(k-2)=2x(k)-x(k-1)+2x(k-2)$ (4 marks)

Question FIVE

(a) Derive the state equations and the transfer function $H(z)=Y(z)/R(z)$ for the block diagram of Figure Q5 (a).

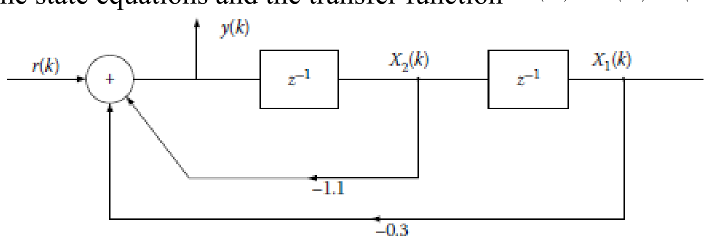


Figure Q5 (a). (8 marks)

(a) Figure Q5 (b) shows a digital control system. When the controller gain K is unity and the sampling time is 0.5 seconds, determine:

i) Open loop pulse transfer function

ii) Closed loop pulse transfer function

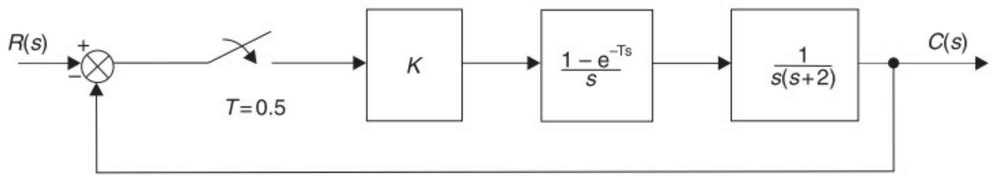


Figure Q5 (b) (12 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k=0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n=k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{\left(1-e^{-aT}\right)z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-aT}z^{-1}\right)}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{\left(e^{-aT}-e^{-bT}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)\left(1-e^{-bT}z^{-1}\right)}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}\left(1+e^{-aT}z^{-1}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{\left[\left(aT-1+e^{-aT}\right)+\left(1-e^{-aT}-aTe^{-aT}\right)z^{-1}\right]z^{-1}}{\left(1-z^{-1}\right)^2\left(1-e^{-aT}z^{-1}\right)}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{\left(1-az^{-1}\right)^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}\left(1+az^{-1}\right)}{\left(1-az^{-1}\right)^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}\left(1+4az^{-1}+a^2z^{-2}\right)}{\left(1-az^{-1}\right)^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}\left(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3}\right)}{\left(1-az^{-1}\right)^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$