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TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Science in Electrical Engineering

EEE 4410: Control Engineering III

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2024 EXAM

TIME: 2 HOURS

DATE: 18TH AUGUST 2024

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five Questions; Attempt Question ONE and any other TWO Questions.

Do not write on the question paper.

Question ONE

(a) State any TWO advantages and two drawbacks of state variable analysis (4 marks)

(b) Define phase variables (1 mark)

(c) Construct a state model for a system characterized by the differential equation,

$$\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+11\frac{dy}{dt}+6y+u=0$$

. Also give the block diagram representation of the state model. (6 marks)

(d) Derive a state space model for the electrical parallel RLC network given in Figure Q1 (c) given that the current $i = I\sin\omega t$. (6 marks)

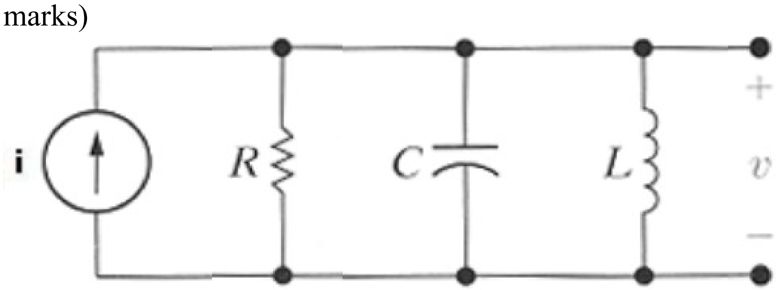

Figure Q1 (d)

(e) Obtain the diagonal canonical form of the state space representation for a system given by:

$$\frac{Y(s)}{U(s)}=\frac{s+3}{s^2+3s+2}$$

(4 marks)

(f) Derive the Transfer Function Matrix for the system described in state space as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -15 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y=\begin{bmatrix}-66 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+[5]u$$

(9 marks)

Question TWO

(a) The state space equation of a control system is given as:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$, with $x_1(0)=1, x_2(0)=0$

Evaluate:

i) The characteristic equation, its roots, ω_n and ζ
ii) The transition matrices $\varphi(s)$ and $\varphi(t)$
iii) The transient response of the state variables from the set of initial conditions.

(16 marks)

(b) Sketch the time response of the state variables together with the state trajectories of the system in (a) (4 marks)

Question THREE

(a) State what is meant by state controllability and state observability (2 marks)
(b) Explain the method of pole placement by state-feedback (2 marks)
(c) Explain the design of state observer. (4 marks)
(d) The plant is described by the differential equation

$$\frac{d^3y}{dt^3}+5\frac{d^2y}{dt^2}+3\frac{dy}{dt}+2y=u$$

Design a state feedback controller for the plant such that the poles of the closed-loop system are located at {-1, -2, -3} assuming that we have access to all states.in state space by the model.

(12 marks)

Question FOUR

(a) State the following Z- transformation theorems:

i) Linearity
ii) Shift theorem
iii) Final value theorem
iv) Initial value theorem

(4 marks)

(b) A sampled data system has the following transfer function: $G(s)=\frac{1}{s+1}$. If the sampling time is 1second and the system is subjected to a unit step input, determine the discrete time response. Note that the system has no zero-order hold included. (6 marks)

(c) Obtain the pulse transfer function $G(z)$ for the system with $G(s)=\frac{1}{(s+1)(s+2)}$ (6 marks)
(d) Explain the main differences between *s*- and *z*- planes (4 marks)

Question FIVE

(a) Derive the state equations and the transfer function $H(z)=Y(z)/R(z)$ for the block diagram of Figure Q5 (a).

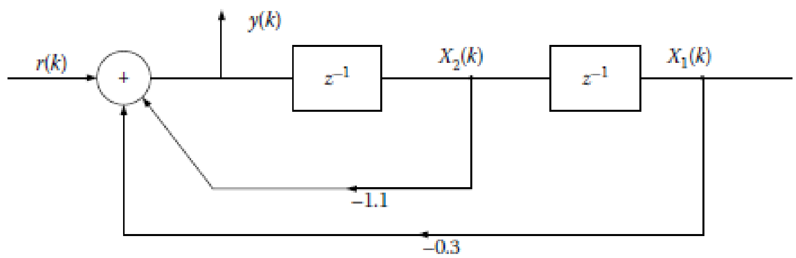

Figure Q5 (a). (8 marks)

(b) For the system of the Fig Q 5(b), if $G(s)=\dfrac{1}{s(s+1)}$ and Let: $T = 0.1$ s:

i) Derive the transfer function of the digital controller $D(z)$ when the desired transfer function of the closed system is

$$G_{cl}(z)=\frac{z+1}{z^2-1.14z+0.403}$$

ii) Derive and design the discrete-time response $y(kT)$ of the closed system when the input is a unit step signal.

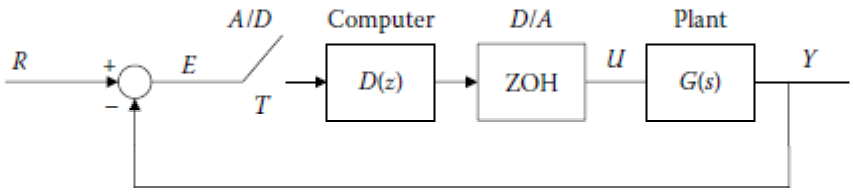

Figure Q5 (b). (12 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0 \quad$ for $t < 0$
$x(kT) = x(k) = 0 \quad$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$