PP1

TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4423: NONLINEAR AND MULTIVARIABLE CONTROL

END OF SEMESTER EXAMINATION

SERIES: APRIL 2025 EXAM

TIME: 2 HOURS

DATE: April 2025

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **five** Questions; Attempt THREE Questions.

Do not write on the question paper.

Question ONE

(a) Distinguish between linear and nonlinear systems (2 marks)

(b) Describe the characteristics of the Saturation nonlinearities (2 marks)

(c) Linearize the nonlinear equation $z = xy$ in the region $5 \le x \le 7; 10 \le y \le 12$ using Taylor series (5 marks)

(d) Briefly explain what is meant by phase plane analysis (2 marks)

(e) State any TWO advantages and TWO disadvantages of phase plane analysis (4 marks)

(f) A first order system is described by $\dot{x} = -4x + x^3$. Determine the singular points and hence sketch the phase trajectory (5 marks)

Question TWO

(a) The differential equation of a certain physical system is given by

$\dddot{y} + 4\ddot{y} + f(y) = 0$

The function $f(y)$ is nonlinear, but it can be approximated by the piecewise-linear graph illustrated in Figure Q2 (a). Determine a piecewise-linear approximation for the nonlinear system differential equation. (5 marks)

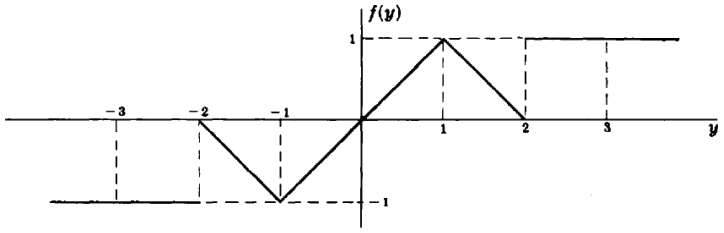

Figure Q2 (a)

(b) Obtain the singular points, the Linearized system matrix A for the singular points and the Eigen values for a system described in state space as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_2 - 2x_1 + x_1^2$$

(9 marks)

(c) Sometimes non-linear elements are intentionally introduced into control system. Give an example stating clearly the reason for the introduction of non-linear element. (2 marks)

(d) Briefly explain the following:
i) Autonomous system
ii) Limit cycles

(4 marks)

Question THREE

(a) Explain the describing function method for nonlinear control system analysis (3 marks)

(b) The characteristics of a hardening spring are given by:

$$f(x) = x + \frac{x^3}{2}$$

Given the input $x = A\sin(\omega t)$ obtain the describing function of the spring (7 marks)

(c) The system is given by:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -x_1(t) + 3x_2^2(t) \\ -x_1(t)x_2(t) - x_2^3(t) \end{bmatrix}$$

Prove that the system is stable according to the function of Lyapunov $V(x) = \frac{1}{2}(x_1^2(t) + x_2^2(t))$ (10 marks)

Question FOUR

(a) Explain the principle of Adaptive control strategy (3 marks)

(b) Distinguish between Model Reference Adaptive Control (MRAC) and Self-Tuning Control (STC) control strategies (4 marks)

(c) Explain Optimal control and discuss any two types of optimal control problems (3 marks)

(d) Obtain the Optimal control Law which minimizes the performance index

$$J = \int_0^\infty (x_1^2 + u^2 t), t)\, dt$$

For the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Given that

$$Q = \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \quad R = 2$$

(10 marks)

Question FIVE

(a) Discuss the relative gain array (RGA) technique as applied to multivariable systems (4 marks)

(b) Derive the transfer function of the system shown in Figure Q 5(b)

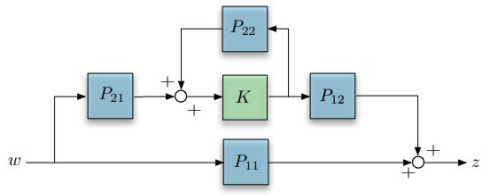

Figure Q 5(b) (8 marks)

(c) Consider a 2× 2 MIMO system having the transfer function:

$$G(s)=\begin{bmatrix} \dfrac{1}{(s+1)(s+2)} & \dfrac{-0.5}{s+1} \\ \dfrac{1}{s+2} & \dfrac{2}{(s+1)(s+2)} \end{bmatrix}$$

Find the poles and zeros performing elementary row and column operations (8 marks)