PP1

TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical Engineering

TEE 4421: STATE SPACE AND DIGITAL CONTROL

END OF SEMESTER EXAMINATION

SERIES: MAY 2025 EXAM

TIME: 2 HOURS

DATE: April 2025

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

A table of Laplace and Z-transforms

This paper consists of **five** Questions; Attempt any THREE Questions.

Do not write on the question paper.

Question ONE

(a) Obtain the state space representation of the system described by: $\frac{d^3y}{dt^3}+6\frac{d^2y}{dt^2}+11\frac{dy}{dt}+6y=6u$ where y is the output and u is the input to the system. (6 marks)

(b) Given the linear single-input, single-output, mass-spring-damper translational mechanical system of Figure Q1 (b), derive the system model and then convert it to a state-space description. For this system, the input is the force $f(t)$ and the output is displacement $y(t)$ (6 marks)

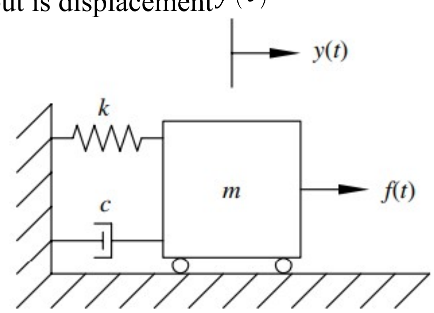

Figure Q1 (b)

(c) Determine the transfer function of a system with a state model given below.

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-2 & -3\\ 4 & 2\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

(8 marks)

Question TWO

(a) State the Shannon's sampling theorem (1 mark)

(b) Explain the sample and hold process. (4 marks)

(c) Figure Q2 (c) shows a digital control system. When the controller gain K is unity and the sampling time is 0.5 seconds, determine:

i) Open loop pulse transfer function
ii) Closed loop pulse transfer function
iii) The difference equation for the discrete time response
iv) A sketch of the unit step response assuming zero initial conditions
v) The steady state value of the system output

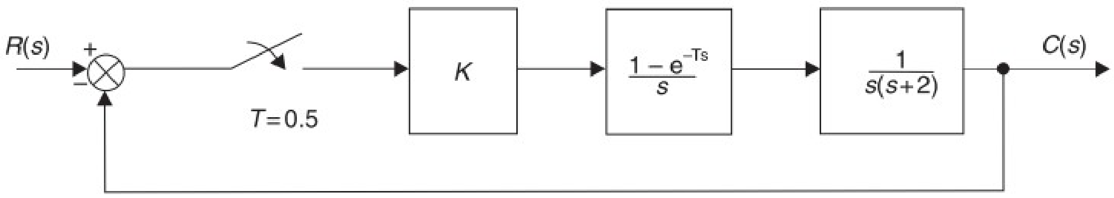

Figure Q2 (c) (15 marks)

Question THREE

(a) A control system has an open loop transfer function:

$$\frac{U(s)}{Y(s)}=\frac{1}{s(s+4)}$$

When $x_1=y$ and $x_2=\dot{x}_1$, express the state equation in the controllable canonical form and evaluate the coefficients of the state feedback gain matrix such that the closed loop poles have the values $s=-2, s=-2$

(10 marks)

(b) A system is described by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}[u]$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Design a full order observer that has an undamped natural frequency of $10\,rad/\sec$ and a damping ratio of 0.5

(10 marks)

Question FOUR

(a) Draw the block diagram and signal flow graph of discrete state-space model (4 marks)

(b) Sketch a block diagram of a discrete-time system with the state space representation

$$x(k+1)=\begin{bmatrix}0 & 1\\ 2 & 5\end{bmatrix}x(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k)$$

$$y(k)=\begin{bmatrix}8 & -7\end{bmatrix}x(k)$$

(4 marks)

(c) Check the stability in both the discrete complex z-plane and the complex s-plane of the sampled data system of figure Q4(c) if $G(s)=\frac{2}{s(s+4)}$, and the sampling period is $T=1sec$. (12 marks)

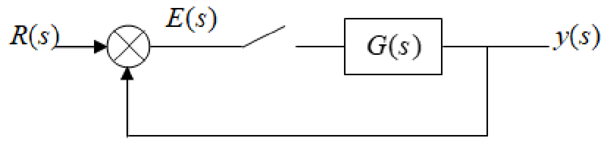

Figure Q4(c)

Question FIVE

(a) Distinguish between a singular and non-singular matrix (2 marks)

(b) The state space equation of a 2[nd] order system is given by:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}[u]$$

Evaluate

i) The characteristic equation, its roots, ω_n and ζ

ii) The transition matrices $\varphi(s)$ and $\varphi(t)$

iii) The transient response of the state variables from the set of initial conditions $y(0)=1$ and $\dot{y}(0)=0$

(18 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$