

DATE: April 2025

Do not write on the question paper.

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- (e) Explain bilinear transformation (8 marks)
 (f) Derive the discrete-time transfer function of the following continuous-time plant. Use a zero-order hold element and assume the sampling time T to be known. (2 marks)
 (7 marks)

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

Question TWO

- (a) State the effects of the following control actions on system performance.
 i) Integral action and
 ii) Derivative action

(4 marks)

- (b) A three-term controller is described by the equation

$$\theta_c(t) = 20 \left[e(t) + \frac{1}{T_r} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \right]$$

Where $e(t)$ = system error, $\theta_c(t)$ = controller output, T_r = reset time and T_d = derivative time. This is used to control a process with transfer function

$$G(s) = \frac{40}{10s^2 + 80s + 800}. \text{ Unity feedback is used.}$$

- i) If integral action is not employed, find the derivative time required to make the closed-loop damping ratio unity.
 ii) If this value of derivative time is maintained. Determine the minimum value of reset time that can be used without instability arising.

(8

marks)

- (c) Closed loop control system shown in Figure Q 2(c) in which a PID controller is used to control the system.

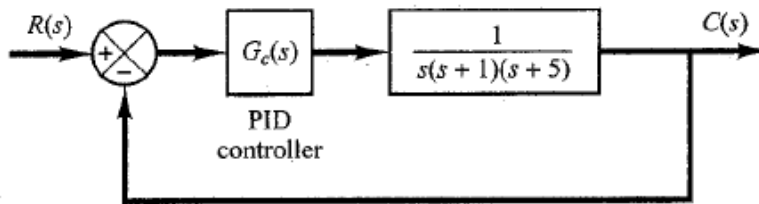


Figure Q 2(c)

The PID controller has the transfer function

$$G_c(s) = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

Apply a Ziegler-Nichols ultimate cycle tuning rule for the determination of the values of parameters K_p , τ_i and τ_d given the following data: $K_p = 0.6 K_c$, $T_i = T_c/2$, $T_d = T_c/8$; The notations have the usual meanings.

(8 marks)

Question THREE

- (a) Briefly describe the phase plane and the describing function techniques for nonlinear system analysis

(8

marks)

- (b) Derive the expression for describing function of the non-linearity function of an ideal relay as shown in figure Q3 (b) given that the output

$$w(t) = \begin{cases} -M, & -\pi \leq \omega T < 0, \\ -M, & 0 \leq \omega T < \pi. \end{cases} \quad (6 \text{ marks})$$

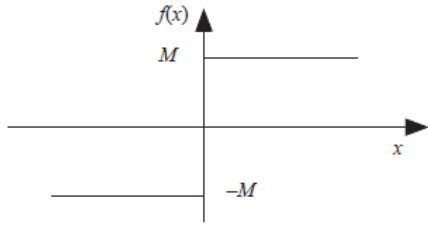


Figure Q3 (b)

- (c) State what is meant by Jump phenomenon in nonlinear systems (2 marks)
- (g) Explain the following nonlinear phenomena (4 marks)
- Finite escape time.
 - Limit circles.

Question FOUR

- (a) Explain Sampling theorem (2 marks)
- (b) Define the initial and final value of the system's impulse when
- $$Y(z) = \frac{2z^2}{(z-1)(z-\alpha)(z-\beta)}, |\alpha|, |\beta| > 1$$
- (4 marks)
- (c) Derive the difference equations of the systems with transfer functions:
- $G(z) = \frac{z^4 + 3z^3 + 2z^2 + z + 1}{z^4 + 4z^3 + 5z^2 + 3z + 2}$ and
 - $H(z) = \frac{z}{5z^2 - 1.7z + 0.72}$ (6 marks)
- (d) Implement digital PID controller algorithm using the position and the velocity versions. (8 marks)

Question FIVE

- (a) Derive the relation between s plane and z plane using Bilinear transformation technique (6 marks)
- (b) The discrete time system is to have the identical time response to the continuous system. State the desired closed loop poles and characteristic equations in
- the s plane and
 - the z plane
- (2 marks)
- (c) Derive the state equations and the transfer function $H(z) = Y(z)/R(z)$ for the block diagram of Figure Q5 (c). (8 marks)

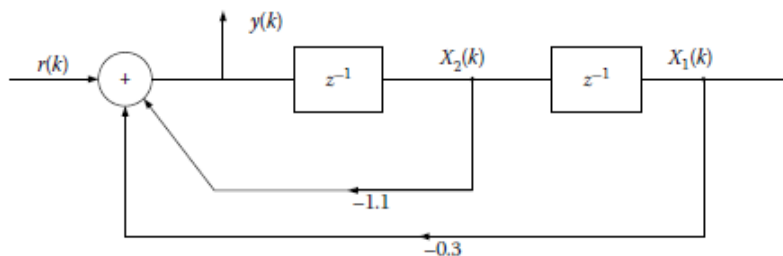


Figure Q5 (c).

- (d) Sketch a block diagram of a discrete - time system with the state space representation as shown

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 8 & -7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(4 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{T e^{-aT} z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT} (1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT}) + (1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1} \cos \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1-e^{-aT} z^{-1} \cos \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.	—	—	a^k	$\frac{1}{1-az^{-1}}$
19.	—	—	a^{k-l} $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	—	—	ka^{k-l}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	—	—	$k^2 a^{k-l}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	—	—	$k^3 a^{k-l}$	$\frac{z^{-1}(1+4az^{-1}+a^2 z^{-2})}{(1-az^{-1})^4}$
23.	—	—	$k^4 a^{k-l}$	$\frac{z^{-1}(1+11az^{-1}+11a^2 z^{-2}+a^3 z^{-3})}{(1-az^{-1})^5}$
24.	—	—	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
 $x(kT) = x(k) = 0$ for $k < 0$
 Unless otherwise noted, $k = 0, 1, 2, 3, \dots$