TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Applied Physics (Control and Instrumentation)

EEE 4450: CONTROL III

END OF SEMESTER EXAMINATION

SERIES: APRIL 2025 EXAM

TIME: 2 HOURS

DATE: April 2025

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

A table of Laplace and Z-transforms

This paper consists of **five** Questions; Question ONE is compulsory. In addition, attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE (Compulsory 20 marks)

(a) Give any THREE advantages of state space approach to control systems analysis. (3 marks)

(b) State what is meant by Phase variables (1 mark)

(c) Find the state space model for the system having transfer function.

$$\frac{Y(s)}{U(s)}=\frac{1}{s^2+s+1}$$

(5 marks)

(d) Calculate the transfer function of the system represented in the state space model as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-1 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u$$

$$y=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

(6 marks)

(e) Determine the state representation of a continuous-time LTI system given below in Jordan canonical form.

$$G(s)=\frac{1}{(s+3)^3(s+2)}$$

(5 marks)

Question TWO

(a) State what is meant by homogenous and non-homogenous state equations (2 marks)

(b) Define state transition matrix. (2 marks)

(c) Find the STM for a system described by $\dot{x}(t)=Ax(t)+Bu(t)$ where

$A=\begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ by the methods of

i) inverse Laplace transforms
ii) series summation method

(12 marks)

(d) Also, determine the solution of the system, that is, State Vector $x(t)$ with input $u(t)=1$ (unit step function) for $t \le 0$ and initial vector $x(0)=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (4 marks)

Question THREE

(a) Obtain the Eigen values and Eigen vectors of $Ax=\lambda x$ given that $A=\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$, and if the matrix A can be diagonalized, give a matrix P such that $P=P^{-1}AP$ is diagonal (7 marks)

(b) Determine whether the system represented by the following state and output equations is completely controllable and observable. (6 marks)

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -2 & -3 & 0 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u$$

$$y=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

(c) For a system whose state space representation is given by:

$$\dot{x}(t)=\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t)+\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

Determine the full state feedback matrix H which places the poles of the closed loop system at positions $s_1=(-2,+j2)$, $s_2=(-2,-j2)$ (7 marks)

Question FOUR

(a) State any Two advantages and Two disadvantages of Digital Control Systems (4 marks)

(b) Find the z-transform of the unit step function $f(t)=1$ (4 marks)

(c) Explain the sampling process. (4 marks)

(d) Given that: $U(z)=\dfrac{z}{(z-2)(z-4)}$, Expand $U(z)$ into partial fraction form and find its time domain representation using inverse z-transform . (8 marks)

Question FIVE

(a) Draw the block diagram and signal flow graph of discrete state-space model (4 marks)

(b) Determine the state equations of a discrete-time LTI system with system function

$$G(z)=\frac{z}{2z^2-3z+1}$$

(6 marks)

(c) Sketch a block diagram of a discrete-time system with the state space representation

$$x(k+1)=\begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} x(k)+\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k)=\begin{bmatrix} 8 & -7 \end{bmatrix} x(k)$$

(4 marks)

(d) A discrete time system is to have the identical time response to the continuous system. State the desired closed loop poles and characteristic equations in

i) the s plane and
ii) the z plane

(4 marks)

(e) State the conditions of stability in the z-domain for the closed loop system of Figure Q 5(e). (2 marks)

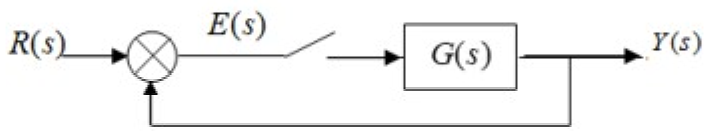

Figure Q 5(e)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k=0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n=k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$