PP1

TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Science in Electrical & Electronic Engineering

EEE 4410: CONTROL ENGINEERING III

END OF SEMESTER EXAMINATION

SERIES: APRIL 2025 EXAM

TIME2 HOURS

DATE: April 2025

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **five** Questions; Question ONE is compulsory. In addition attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE

(a) State what is meant by state variables of a system (2 marks)

(b) Obtain the transfer function for the system from the state-space equations.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(6 marks)

(c) Obtain the diagonal canonical state space representation of a feedback system having a closed - loop transfer function

$$\frac{Y(s)}{U(s)} = \frac{2(s+5)}{(s+2)(s+3)(s+4)}$$

(6 marks)

(d) Obtain the state-space model of the electrical circuit shown in Figure Q 1(d). (6 marks)

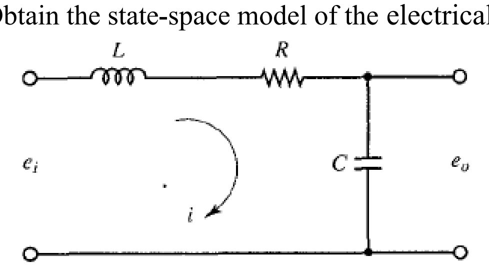

Figure Q 1(d)

(e) Briefly, explain the sampling theorem (2 marks)
(f) Describe the Sample & Hold operation (4 marks)
(g) Find the z-transform of a unit ramp signal (4 marks)

Question TWO

(a) State what is meant by Eigen values and Eigen vectors of a matrix (2 marks)
(b) Obtain the Eigen values and Eigen vectors of $Ax=\lambda x$ given that
$A=\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix}$, and if the matrix A can be diagonalized, give a matrix P such that $P=P^{-1}AP$ is diagonal
(8 marks)
(c) State what is meant by Bilinear transformation (2 marks)
(d) Derive the relation between s plane and z plane using Bilinear transformation technique (6 marks)
(e) The discrete time system is to have the identical time response to the continuous system. State the desired closed loop poles and characteristic equations in
i) the s plane and
ii) the z plane
(2 marks)

Question THREE

(a) The state space system is given
$$\dot{x}_1(t)=2x_1+3x_2$$
$$\dot{x}_2(t)=x_1+4x_2+u(t)$$
$$y(t)=x_1+x_2, \qquad x(0)=\begin{bmatrix}1\\0\end{bmatrix}$$
Where $u(t)$ is a unit step $u(t)=1(t),\ t\geq 0$
Find:
i) The first component of the solution for state space vector in time domain.
ii) The second component of the solution for state space vector in time domain
iii) The output of the system
(16 marks)
(b) Verify the controllability and observability of a control system which is represented in the state space model as,
$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-1 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$
(4 marks)

Question FOUR

(a) A control system has an open loop transfer function:
$$\frac{Y(s)}{U(s)}=\frac{1}{s(s+4)}$$
When $x_1=y$ and $x_2=\dot{x}_1$, express the state equation in the controllable canonical form and evaluate the coefficients of the state feedback gain matrix such that the closed loop poles have the values $s=-2, s=-2$
(10 marks)
(b) Let $A=\begin{bmatrix}1/2 & 1\\ 1 & 2\end{bmatrix}$, $B=\begin{bmatrix}1\\1\end{bmatrix}$, and the desired eigenvalues be $-1\pm j$.

Determine the closed loop gain matrix K. using Ackermann's formula (5 marks)

(c) Consider the system

$\dot{x}(t) = Ax + Bu(t)$

$y(t) = Cx(t)$

Where, $A = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, and\, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$

Design a full order state observer assuming that the desired Eigen values of the observer gain matrix are $\mu_1 = -10, \mu_2 = -10$ (5 marks)

Question FIVE

(a) A sampled data system has the following transfer function: $G(s) = \dfrac{1}{s+1}$. If the sampling time is 1second and the system is subjected to a unit step input, determine the discrete time response. Note that the system has no zero-order hold included. (6 marks)

(b) Derive the state equations and the transfer function $H(z) = Y(z)/R(z)$ for the block diagram of Figure Q5 (b).

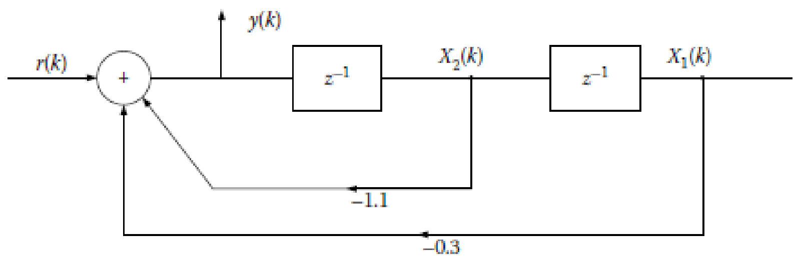

Figure Q5 (b). (10 marks)

(c) Sketch a block diagram of a discrete - time system with the state space representation as shown

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 8 & -7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(4 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k\cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$