TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4423: NONLINEAR AND MULTIVARIABLE CONTROL

END OF SEMESTER EXAMINATION

SERIES: JULY 2025 EXAM

TIME: 2 HOURS

DATE: July 2025

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **five** Questions; Attempt THREE Questions.

Do not write on the question paper.

Question ONE

(a) Explain what is meant by jump resonance phenomenon in nonlinear systems (2 marks)

(b) Explain with sketches the following types of nodes (6 marks)

i) Stable node

ii) Saddle point

iii) Stable focus.

(c) With the aid of the input / output characteristic, explain the significance of saturation phenomenon (2 marks)

(d) Linearize the nonlinear equation $z = xy$ in the region $5 \leq x \leq 7; 10 \leq y \leq 12$ using Taylor series (6 marks)

(e) Briefly explain the following:

i) Autonomous system

ii) Limit cycles

(4 marks)

Question TWO

(a) Describe the following nonlinear stability analysis tools

i) phase plane method

ii) describing function technique

(8 marks)

(b) A nonlinear system is described in state space by the state equation

$\dot{x}_1 = ax_1 - x_1 x_2$

$\dot{x}_2 = bx_1{}^2 + cx_2$

with a, b, and c being positive constants

i) Determine all equilibrium points of the system

ii) Compute the jacobian of the system
iii) Determine the type of each equilibrium point.

(12 marks)

Question THREE

(a) Explain the Lyapunov direct method of nonlinear system analysis (4 marks)

(b) The system is given by:

$$\dot{x}(t)=\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-x_1(t)+3x_2^2(t)\\ -x_1(t)x_2(t)-x_2^3(t)\end{bmatrix}$$

Prove that the system is stable according to the function of Lyapunov $V(x)=\frac{1}{2}(x_1^2(t)+x_2^2(t))$ (10 marks)

(c) A mass-spring system is given in Figure Q 3(c). Derive the governing equation and hence sketch the system phase trajectories for different initial conditions. (6 marks)

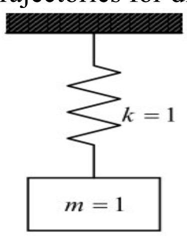

Figure: Q 3 (c).

Question FOUR

(a) Explain Nyquist Stability theorem when applied to MIMO Systems (4 marks)
(b) Discuss the relative gain array (RGA) technique as applied to multivariable systems (4 marks)
(c) Determine the Relative gain array (RGA) matrix of the following MIMO system

$$P(s)=\begin{bmatrix}\frac{-1}{s+1} & \frac{2}{s+1}\\ \frac{s}{s+3} & \frac{2s+1}{s+3}\end{bmatrix}$$

(8 marks)

(d) Briefly state the concept of the Gershgorin's Circle Theorem (4 marks

Question FIVE

(a) State Optimal control strategy (2 marks)
(b) Explain the following optimal control policies:
i) Terminal control problem
ii) Minimum time control problem
iii) Minimum energy control problem
iv) The regulator control problem
v) The tracking control problem

(5 marks)

(c) Determine the optimum trajectory $x(t)$ which minimizes the cost function

$$J=\int_0^{\pi/2}[x^2(t)+\dot{x}^2(t)]dt$$

With boundary conditions $x(t_0)=x(0)=0$ and $x(t_f)=x(\pi/2)=1$ (9 marks)

(d) Distinguish between Model Reference Adaptive Control (MRAC) and Self-Tuning Control (STC) control strategies (4 marks)