TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical Engineering

TEE 4421: STATE SPACE AND DIGITAL CONTROL

END OF SEMESTER EXAMINATION

SERIES: JULY 2025 EXAM

TIME: 2 HOURS

DATE: July 2025

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

A table of Laplace and Z-transforms

This paper consists of **five** Questions; Attempt any THREE Questions.

Do not write on the question paper.

Question ONE

(a) Compare and contrast classical control to modern control approach to control systems analysis. (4 marks)

(b) Figure Q1 (b) is a series RLC electrical circuit. It is having an input voltage, $V_i(t)$ and the current flowing through the circuit is $i(t)$. Develop the state space representation of the circuit if the state variables are the current flowing through the inductor, $i(t)$ and the voltage across capacitor, $V_o(t)$. (6 marks)

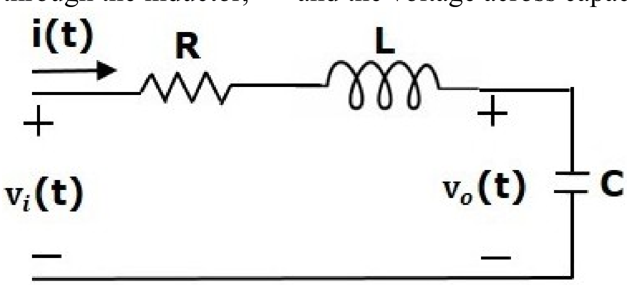

Figure Q1 (b)

(c) The dynamics of three cars moving can be represented in state space by the following state equations, where u(t) is an input control for the leading car

$$\dot{x}_1(t)=x_2(t)-x_1(t)+u(t)$$

$$\dot{x}_2(t)=\frac{x_1(t)+x_3(t)}{2}-x_2(t)$$

$$\dot{x}_3(t)=x_2(t)-x_3(t)$$

Represent the above dynamics in the form $\dot{x}(t)=Ax+Bu(t)$ (4 marks)

(d) Determine the state representation of a continuous-time LTI system given below in Jordan canonical form.

$$G(s)=\frac{1}{(s+3)^3(s+2)}$$

(6 marks)

Question TWO

(a) State the Shannon's sampling theorem (2 marks)

(b) Given that:

$U(z)=\frac{z}{(z-2)(z-4)}$, Expand $U(z)$ into partial fraction form and find its time domain representation using long division method . (8 marks)

(c) Discretize the continuous-time system given in the form $x(k+1)=A_d x(kT)+B_d u(kT)$ and deduce the expressions for the matrices A_d and B_d for the sampling period $T=10\,\text{sec}$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -0.01 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

(10 marks)

Question THREE

(a) Explain the terms controllability and observability (4 marks)

(b) A system is described in state space by: $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 1\\ 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u$. Determine whether the system is completely state controllable or not. (4 marks)

(c) A regulator system is shown in Figure Q3(c). The plant is given by: $\dot{x}=Ax+Bu$ where,

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -5 & -6\end{bmatrix},\quad B=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}.$$

The system uses the state feedback control $u=-Kx$. The desired closed-loop poles are chosen to be at s = -2 + j4, s = -2 - j4, s = -10. Determine the state feedback gain matrix K.

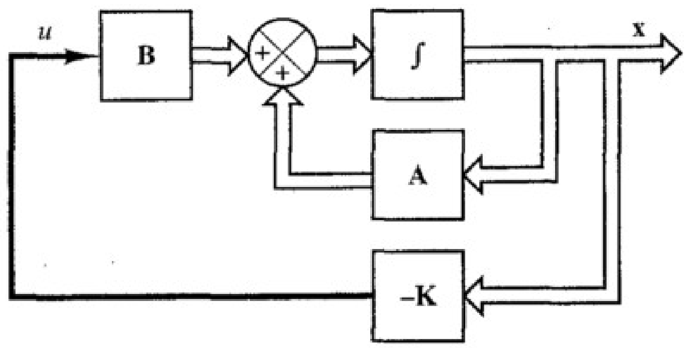

Figure Q3 (c) (12 marks)

Question FOUR

(a) Determine the z- transforms for the following continuous functions:

i) step function $f_1(t)=u(t)=1(t)$

ii) ramp input $f_2(t)=at$

(6 marks)

(b) Draw the block diagram and signal flow graph of discrete state-space model (4 marks)

(a) Sketch a block diagram of a discrete-time system with the state space representation

$$x(k+1)=\begin{bmatrix}0 & 1\\ 2 & 5\end{bmatrix}x(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k)$$

$$y(k)=\begin{bmatrix}8 & -7\end{bmatrix}x(k)$$

(4 marks)

(c) For the transfer function $G(s)=\dfrac{1}{(s+1)(s+2)}$ obtain the pulse transfer function $G(z)$ (6 marks)

Question FIVE

(a) Obtain a state-space model in the phase variable form for a system with transfer function

$$\frac{Y(s)}{U(s)}=\frac{s^3+7s^2+5s+2}{s(s+1)(s+3)}$$

(4 marks)

(b) For the same system obtain a state-space model in the canonical form. (4 marks)

(c) Show that the eigen values of A matrix in (b) and (c) are 0, -1 and -3 (4 marks)

(d) Apply e^{At} method to determine the time response of a system described by the homogeneous equation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

given that $x_1(0)=1$ and $x_2(0)=1$

(8 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $\quad k = 0$ 0 $\quad k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $\quad n = k$ 0 $\quad n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$