TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4423: NONOLINEAR AND MULTIVARIABLE CONTROL

END OF SEMESTER EXAMINATION

SERIES: NOVEMBER 2024 EXAM

TIME: 2 HOURS

DATE: Pick DateSelect MonthPick Year

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of five Questions; Attempt any THREE Questions.

Do not write on the question paper.

Question ONE

(a) Distinguish between linear and nonlinear systems (2 marks)

(b) Explain various types of singular points encountered in non-linear systems and draw approximately the phase plane trajectories of these points. (12 marks)

(c) Draw the phase trajectory for the system described by the following differential equation

$\frac{d^2x}{dt^2}+0.6\frac{dx}{dt}+x=0$. With $x(0)=1$ and $\frac{dx}{dt}(0)=0$ (6 marks)

Question TWO

(a) Explain the following nonlinear phenomena

i) Finite escape time.

ii) Multiple isolated equilibria.

iii) Limit circles.

iv) Bifurcation

(8 marks)

(b) Linearize the nonlinear equation $z = xy$ in the region $5 \le x \le 7; 10 \le y \le 12$ using Taylor series (5 marks)

(c) Explain what is meant by an autonomous system (2 marks)

(d) Find the singular points of the pair of equations

$\dot{x}_1 = \sin x_2$

$\dot{x}_2 = x_1 + x_2$

(5 marks)

Question THREE

(a) Explain the concepts of Phase Plane Analysis (3 marks)

(b) A mass-spring system is given in Figure Q 3(b). Derive the governing equation and hence sketch the system phase trajectories for different initial conditions. (7 marks)

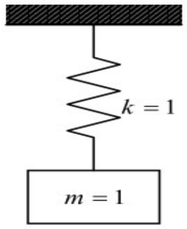

Figure: Q 3 (b).

(c) Explain the concept of describing function techniques for the analysis of nonlinear systems (3 marks)

(d) Given a nonlinear system $y(t)=x^2\frac{dx}{dt}+2x$, where x denotes the input and y is the output, derive the describing function for an input $x=X\sin(\omega t)$ (7 marks)

Question FOUR

(a) Briefly explain what is meant by the term adaptive control (3 marks)

(b) State Optimal control strategy (2 marks)

(c) Explain the following optimal control minimization policies:

i) Terminal control problem
ii) Minimum time control problem
iii) Minimum energy control problem
iv) The regulator control problem
v) The tracking control problem

(5 marks)

(d) Determine the optimum trajectory $x(t)$ which minimizes the cost function

$$J=\int_0^{\pi/2}[x^2(t)+\dot{x}^2(t)]dt$$

With boundary conditions $x(t_0)=x(0)=0$ and $x(t_f)=x(\pi/2)=1$ (10 marks)

Question FIVE

(a) Describe the Lyapunov's direct method of stability analysis (3 marks)

(b) Determine sufficient conditions for the stability of the system $\dot{x}=Ax+bf(x_1)$ where:

$A=\begin{bmatrix}-2 & -1\\ 0 & -2\end{bmatrix}, b=\begin{bmatrix}1\\ 2\end{bmatrix}$ given the Lyapunov function $V=x^TPx$ and $P=\begin{bmatrix}a & c\\ c & 1\end{bmatrix}$ (9 marks)

(c) Derive the transfer function of the system shown in Figure Q 5(c)

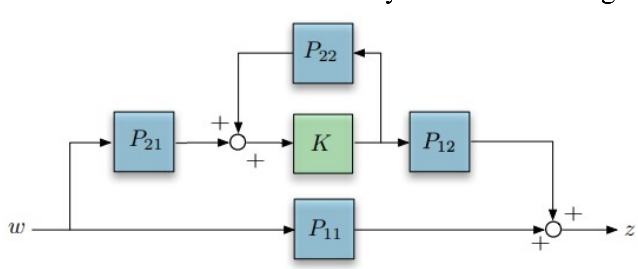

Figure Q 5(c) (8 marks)