PP1

# TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

## UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4423: NONOLINEAR AND MULTIVARIABLE CONTROL

END OF SEMESTER EXAMINATION

**SERIES:** AUGUST 2024 EXAM

**TIME:** 2 HOURS

**DATE:** Pick DateSelect MonthPick Year

**Instructions to Candidates**

You should have the following for this examination

*-Answer Booklet, examination pass and student ID*

This paper consists of **five** Questions; Attempt any THREE Questions.

**Do not write on the question paper.**

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**Question ONE**

(a) Distinguish between Inherent and Intentional nonlinearities (2 marks)

(b) With the aid of the input - output characteristic, explain the significance of saturation phenomenon (2 marks)

(c) Explain what is meant by limit cycles in nonlinear systems (2 marks)

(d) For the system described by the equation given below, construct the trajectory starting at the initial point $(1,0)$ using delta method.

$\ddot{x}+\dot{x}+x^2=0$ (8 marks)

(e) Linearize the nonlinear equation: $z=xy$ in the region $5\le x\le 7$, $10\le y\le 12$. Find the error if the linearized equation is used to calculate the value of $z$ when $x=5, y=10$. (6 marks)

**Question TWO**

(a) The differential equation of a certain physical system is given by: $\dddot{y}+4\ddot{y}+f(y)=0$. The function $f(y)$ is nonlinear, but it can be approximated by the piecewise-linear graph illustrated in Figure Q 2 (b). Determine a piecewise-linear approximation for the nonlinear system differential equation. (5 marks)

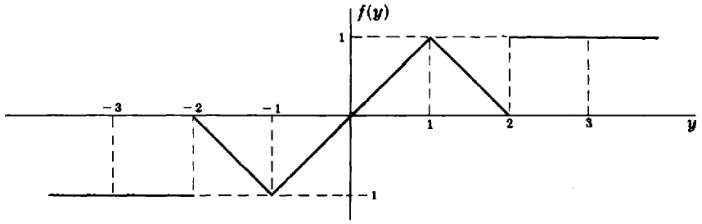


Figure Q2 (b)

(b) State any two advantages and one disadvantage of phase plane analysis (3 marks)

(c) A mass-spring system is given in Figure Q2 (c). Derive the governing equation in state space and hence sketch the system phase trajectories for different initial conditions. (6 marks)

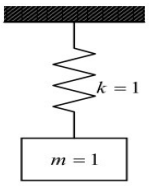


Figure: Q 2(c).

(d) Explain what is meant by an autonomous system (2 marks)

(e) Explain with sketches the following types of nodes (6 marks)

i) Unstable focus
ii) Stable focus.

**Question THREE**

(a) A nonlinear system is described in state space by the state equation

$\dot{x}_1 = ax_1 - x_1 x_2$

$\dot{x}_2 = bx_{1^2} + cx_2$

with a, b, and c being positive constants

i) Determine all equilibrium points of the system
ii) Compute the jacobian of the system
iii) Determine the type of each equilibrium point.

(10 marks)

(b) Explain the merits of describing function technique for the analysis of nonlinear systems (3 marks)

(f) The characteristics of a hardening spring are given by:

$$f(x) = x + \frac{x^3}{2}$$

Given the input $x = A\sin(\omega t)$ obtain the describing function of the spring (7 marks)

**Question FOUR**

(a) Explain Lyapunov stability criterion (3 marks)

(b) The system is given by:

$$\begin{matrix}\dot{x}_1(t) \\ \dot{x}_2(t)\end{matrix} = \begin{matrix}-x_1(t) + 3x_2^2(t) \\ -x_1(t)x_2(t) - x_2^3(t)\end{matrix}$$

Prove that the system is stable according to the function of Lyapunov: $V(x) = \frac{1}{2}(x_1^2(t) + x_2^2(t))$ (7 marks)

(c) State Optimal control strategy (2 marks)

(d) Obtain the Optimal control Law $u_{\text{opt}}$ which minimizes the performance index

$J = \int_0^\infty (x_{1^2} + u^2 t), t)dt$

For the system

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u$$

Given that

$$Q = \begin{bmatrix}0 & 2 \\ 0 & 0\end{bmatrix} \quad R = 2$$

(8 marks)

**Question FIVE**

(a) Discuss the relative gain array(RGA) technique as applied to multivariable systems (2 marks)

(b) Derive the transfer function of the system shown in Figure Q 5(c)

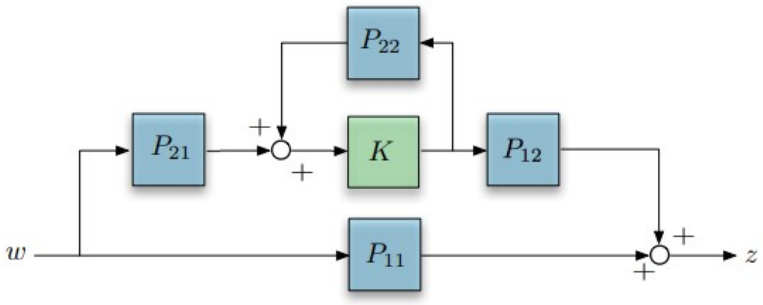


Figure Q 4(b) (8 marks)

(c) Briefly explain what is meant by the term adaptive control and hence distinguish between model reference adaptive control and self-tuning adaptive control. (10 marks)