PP2

# TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

## UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4421: STATE SPACE DESIGN AND DIGITAL CONTROL

END OF SEMESTER EXAMINATION

**SERIES:** NOVEMBER 2024 EXAM

**TIME:** 2 HOURS

**DATE:**Pick DateSelect MonthPick Year

**Instructions to Candidates**

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **five** Questions; Question ONE is compulsory. In addition attempt any Other TWO Questions.

**Do not write on the question paper.**

**Question ONE**

(a) Define state of a system, state variables, state space and state vector and state the advantages of state space analysis (6 marks)

(b) A feedback system has a closed loop transfer function:

(c) $\frac{U(s)}{Y(s)} = \frac{10s+40}{s^3+s^2+3s}$. Construct three different state models showing block diagram in each case (12 marks)

(d) Distinguish between singularity and non-singularity matrix (2 marks)

**Question TWO**

(a) Compute $e^{At}$ for $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ (6 marks)

(b) Find the transfer function relating the input $u(t)$ and the output response $y(t)$ for system.

$$\dot{x} = Ax + Bu = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -4 & -2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = Cx + Du = \begin{bmatrix} 5 & 1 & 0 \end{bmatrix} x$$

(8 marks)

(c) Determine the characteristic polynomial and eigenvalues for the systems represented by the dynamics matrix

$$A=\begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$$

(6 marks)

**Question THREE**

(a) Briefly describe an advantage that state-space techniques have over root locus techniques in the placement of closed-loop poles for transient response design. (2 marks)

(b) In order to affect a complete observer design, a system must be observable. Describe the physical meaning of observability. (2 marks)

(c) Given the plant $\begin{matrix}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{matrix}$ where, $A=¿[0 \quad 20.6¿]¿¿¿$ ¿ . Design a full order state observer assuming that the desired Eigen value of the observer matrix are $\mu_{1,2}=-10$ (8 marks)

(d) The state equation of the closed loop system is given by $\dot{x}(t)=(A-BK)x(t)$ where,

$$A=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix},\ K=[6 \quad 1]$$

Check whether the closed loop system is stable using the function of Lyapunov

$$V(x)=\frac{1}{2}x^T(t)Px(t)=\frac{1}{2}x^T\begin{bmatrix}5 & 2\\ 2 & 10\end{bmatrix}x(t)$$

(8 marks)

**Question FOUR**

(a) Briefly, explain the sampling theorem and the Sample & Hold operation (6 marks)

(b) Find the z-transform of a sampled unit ramp signal (6 marks)

(c) Consider the continuous-time plant with transfer function $G_P(s)=\frac{1}{s+1}$. Determine the equivalent discrete-time transfer function for the plant in series with a zero-order hold. (8 marks)

**Question FIVE**

(a) Use long division to determine the first few terms of the impulse response for the transfer function

$$G(z)=\frac{z-4}{(z-3)(z-2)}$$

(6 marks)

(b) Find the equivalent transfer function for the following state variable model

$$\begin{bmatrix}x_1(k+1)\\ x_2(k+1)\end{bmatrix}=\begin{bmatrix}0.1 & 0\\ 0.2 & 0.3\end{bmatrix}\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u(k)$$

$$y(t)=[0 \quad 1]\begin{bmatrix}x_1(k)\\ x_2(k)\end{bmatrix}$$

(6 marks)

(c) Consider the following discrete-time state variable system,

$$x(k+1)=Gx(k)+Hu(k)$$
$$y(k)=Cx(k)$$

Where,

$G=\begin{bmatrix}0.1 & 0.2\\ 0 & 0.2\end{bmatrix}$, $H=\begin{bmatrix}0\\ 0.1\end{bmatrix}$. Find the gain vector $K$ to place the closed loop poles at - 0.1 and -0.2. (8 marks)

## Table of Laplace and Z-transforms

| | $X(s)$ | $x(t)$ | $x(kT)$ or $x(k)$ | $X(z)$ |
|---|---|---|---|---|
| 1. | – | – | Kronecker delta $\delta_0(k)$<br>1 $k = 0$<br>0 $k \neq 0$ | 1 |
| 2. | – | – | $\delta_0(n-k)$<br>1 $n = k$<br>0 $n \neq k$ | $z^{-k}$ |
| 3. | $\frac{1}{s}$ | $1(t)$ | $1(k)$ | $\frac{1}{1-z^{-1}}$ |
| 4. | $\frac{1}{s+a}$ | $e^{-at}$ | $e^{-akT}$ | $\frac{1}{1-e^{-aT}z^{-1}}$ |
| 5. | $\frac{1}{s^2}$ | $t$ | $kT$ | $\frac{Tz^{-1}}{(1-z^{-1})^2}$ |
| 6. | $\frac{2}{s^3}$ | $t^2$ | $(kT)^2$ | $\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$ |
| 7. | $\frac{6}{s^4}$ | $t^3$ | $(kT)^3$ | $\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$ |
| 8. | $\frac{a}{s(s+a)}$ | $1-e^{-at}$ | $1-e^{-akT}$ | $\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$ |
| 9. | $\frac{b-a}{(s+a)(s+b)}$ | $e^{-at}-e^{-bt}$ | $e^{-akT}-e^{-bkT}$ | $\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$ |
| 10. | $\frac{1}{(s+a)^2}$ | $te^{-at}$ | $kTe^{-akT}$ | $\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$ |
| 11. | $\frac{s}{(s+a)^2}$ | $(1-at)e^{-at}$ | $(1-akT)e^{-akT}$ | $\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$ |
| 12. | $\frac{2}{(s+a)^3}$ | $t^2e^{-at}$ | $(kT)^2e^{-akT}$ | $\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$ |
| 13. | $\frac{a^2}{s^2(s+a)}$ | $at-1+e^{-at}$ | $akT-1+e^{-akT}$ | $\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$ |
| 14. | $\frac{\omega}{s^2+\omega^2}$ | $\sin \omega t$ | $\sin \omega kT$ | $\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$ |
| 15. | $\frac{s}{s^2+\omega^2}$ | $\cos \omega t$ | $\cos \omega kT$ | $\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$ |
| 16. | $\frac{\omega}{(s+a)^2+\omega^2}$ | $e^{-at}\sin \omega t$ | $e^{-akT}\sin \omega kT$ | $\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$ |
| 17. | $\frac{s+a}{(s+a)^2+\omega^2}$ | $e^{-at}\cos \omega t$ | $e^{-akT}\cos \omega kT$ | $\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$ |
| 18. | – | – | $a^k$ | $\frac{1}{1-az^{-1}}$ |
| 19. | – | – | $a^{k-1}$<br>$k = 1, 2, 3, \ldots$ | $\frac{z^{-1}}{1-az^{-1}}$ |
| 20. | – | – | $ka^{k-1}$ | $\frac{z^{-1}}{(1-az^{-1})^2}$ |
| 21. | – | – | $k^2a^{k-1}$ | $\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$ |
| 22. | – | – | $k^3a^{k-1}$ | $\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$ |
| 23. | – | – | $k^4a^{k-1}$ | $\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$ |
| 24. | – | – | $a^k\cos k\pi$ | $\frac{1}{1+az^{-1}}$ |

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$