



TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Electrical & Electronic Engineering

TEE 4421: STATE SPACE DESIGN AND DIGITAL CONTROL

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2024 EXAM

TIME: 2 HOURS

DATE: Pick Date Select Month Pick Year

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

This paper consists of **five** Questions; Question ONE is compulsory. In addition attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE

- (a) State any Two Advantages and Two drawbacks of state variable analysis. (4 marks)
- (b) Consider the system shown in Figure Q1 (b) which is series RLC circuit. It is having an input voltage $v_i(t)$ and the current flowing through the circuit is $i(t)$. The state variables are the current through the inductor, and the voltage across the capacitor $v_c(t)$. Represent the circuit as a state space model.

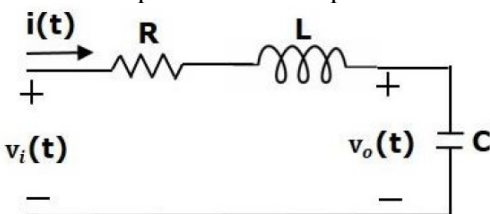


Figure Q1 (b)

- (c) Distinguish between singularity and non-singularity matrix (6 marks)
- (d) Find all eigenvalues and their Eigen vectors for (2 marks)

$$A = \begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix},$$

(8 marks)

Question TWO

- (a) Find the matrix transfer function relating the input $u(t)$ and the output response $y(t) = x_1(t)$ for the second order system.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(6 marks)

- (b) A linear time invariant system is characterized by state equation

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Compute the solution of state equation in time domain, assuming initial vector is

$$\begin{bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(8 marks)

- (c) Obtain the diagonal canonical state space representation of a feedback system having a closed - loop transfer function

$$\frac{U(s)}{Y(s)} = \frac{2(s+5)}{(s+2)(s+3)(s+4)}$$

(6 marks)

Question THREE

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

- (a) Given a plant: $y(t) = Cx(t)$ where, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$
Determine:

- Full state feedback in such a way that the closed loop system has double negative pole $p_{1,2} = -1$
- Full state observer such that its poles are double and are $p_{3,4} = -3$

(12 marks)

- (b) The state equation of the closed loop system is given by $\dot{x}(t) = (A - BK)x(t)$ where,

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, K = \begin{bmatrix} 6 & 1 \end{bmatrix}$$

Check whether the closed loop system is stable using the function of Lyapunov

$$V(x) = \frac{1}{2} x^T(t) P x(t) = \frac{1}{2} x^T \begin{bmatrix} 5 & 2 \\ 2 & 10 \end{bmatrix} x(t)$$

(8 marks)

Question FOUR

- (a) Briefly, explain the sampling theorem and the Sample & Hold operation (6 marks)

- (b) Find the z-transform of a sampled unit ramp signal (6 marks)

- (c) Given a (ZOH) in cascade with

$$G_1(s) = \frac{(s+2)}{(s+1)} \quad G(s) = \frac{(1-e^{-Ts})}{s} \frac{(s+2)}{(s+1)}$$

or . Find the sampled-data transfer function, $G(z)$, if the sampling time, T , is 0.5 second. (8 marks)

Question FIVE

- (a) Given that: $U(z) = \frac{z}{(z-2)(z-4)}$, Expand $U(z)$ into partial fraction form and find its time domain representation using inverse z-transform. (6 marks)

- (b) Check the stability of the system of figure Q 5(b) $G(s) = \frac{2}{s(s+4)}$, where the sampling period is $T=1\text{sec}$

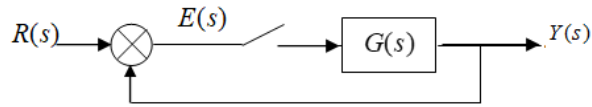


Figure Q 5(b) (10 marks)

- (c) Draw a basic block diagram used in representing the discrete state-space model (2 marks)
- (d) Determine the state equations of a discrete-time LTI system with system function

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

(2 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Tze^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
15.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1} \cos \omega T}{1-2z^{-1} \cos \omega T + z^{-2}}$
16.	$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1-e^{-aT} z^{-1} \cos \omega T}{1-2e^{-aT} z^{-1} \cos \omega T + e^{-2aT} z^{-2}}$
18.	—	—	a^k	$\frac{1}{1-az^{-1}}$
19.	—	—	a^{k-l} $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	—	—	ka^{k-l}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	—	—	$k^2 a^{k-l}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	—	—	$k^3 a^{k-l}$	$\frac{z^{-1}(1+4az^{-1}+a^2 z^{-2})}{(1-az^{-1})^4}$
23.	—	—	$k^4 a^{k-l}$	$\frac{z^{-1}(1+11az^{-1}+11a^2 z^{-2}+a^3 z^{-3})}{(1-az^{-1})^5}$
24.	—	—	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
 $x(kT) = x(k) = 0$ for $k < 0$
 Unless otherwise noted, $k = 0, 1, 2, 3, \dots$