TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Science in Electrical & Electronic Engineering

EEE 4410: CONTROL ENGINEERING III

END OF SEMESTER EXAMINATION

SERIES: JULY 2025 EXAM

TIME: 2 HOURS

DATE: July 2025

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

This paper consists of **five** Questions; Question ONE is compulsory. In addition attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE

(a) Distinguish between classical methods and the modern methods of studying control systems (2 marks)

(b) Distinguish between singular and non-singular matrices (2 marks)

(c) Obtain a state-space model in the phase variable form for a system with transfer function

$$\frac{Y(s)}{U(s)}=\frac{s^3+7s^2+5s+2}{s(s+1)(s+3)}$$

(4 marks)

(d) For the same system obtain a state-space model in the canonical form. (4 marks)

(e) Show that the eigen values of *A* matrix in (b) and (c) are 0, -1 and -3 (4 marks)

(f) Determine the z-transform of $f(t)=e^{-at}$ for $t>0$ *f(t)* (4 marks)

(g) Briefly, explain the sampling theorem (2 marks)

(h) Describe the Sample & Hold operation (4 marks)

(i) Find inverse *z* transform of

$$\frac{z}{(z^2-3z+2)(z-4)}$$

(4 marks)

Question TWO

(a) Figure Q 2 (a) is an electrical network with initial conditions $i_L(0)=1$ and $v_C(0)=0$. If the input is $v(t)$, the state variables are $x_1=\int i\,dt$, $x_2=i_L$ and the output of the network is $y=v_R$. Determine:

i) A state-space description

ii) The state transition matrix

iii) The solution of the homogeneous equation

iv) The general solution

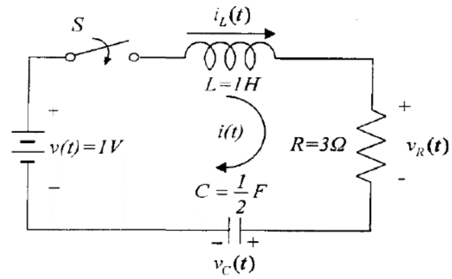

Figure Q 2 (a)

(16 marks)

(b) The dynamics of three cars moving can be represented in state space by the following state equations, where u(t) is an input control for the leading car

$$\dot{x}_1(t)=x_2(t)-x_1(t)+u(t)$$
$$\dot{x}_2(t)=\frac{x_1(t)+x_3(t)}{2}-x_2(t)$$
$$\dot{x}_3(t)=x_2(t)-x_3(t)$$

Represent the above dynamics in the form $\dot{x}(t)=Ax+Bu(t)$ (4 marks)

Question THREE

(a) State the concepts of controllability and observability with regards to multivariable systems (4 marks)

(b) Verify the controllability and observability of a control system which is represented in the state space model as,

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}-1 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

(4 marks)

(c) Consider a system with the transfer function $\frac{Y(s)}{U(s)}=G(s)=\frac{1}{s^2}$. Design the state variable feedback matrix K such that the desired closed loop poles of the system can be placed at $s=-1\pm j$ (6 marks)

(d) A second order system is described in state space as

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$
$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

Design the observer gain matrix G so that the desired damping factor $\zeta=0.8$ and the undamped natural frequency $\omega_n=10\,rad/s$

(6 marks)

Question FOUR

(a) Check the stability of the closed loop system of Figure Q4(a) in complex s-plane and in discrete complex z-plane with $G(s)=\frac{2}{s(s+4)}$, where the sampling period is $T=1sec$ (10 marks)

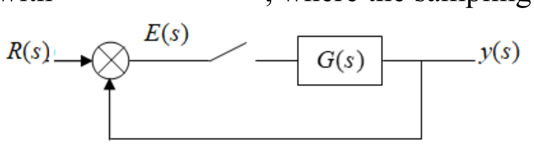

Figure Q4(a)

(b) For the continuous time system $\dot{x} = Ax + Bu$, $A = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, find the sampled data equivalent system representation for the sampling period $T = 10\,\text{sec}$ (10 marks)

Question FIVE

(a) Sketch a block diagram of a discrete - time system with the state space representation as shown

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 8 & -7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(4 marks)

(b) Obtain the pulse transfer function $\frac{Y(z)}{X(z)}$ for the system represented in Figure Q5 (b) (8 marks)

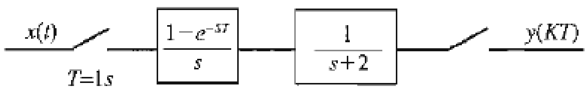

Figure Q5 (b)

(c) Given that

$$Y(z) = \frac{z+1}{z^2 - 1}$$

i) Determine $y(kT)$ for $k = 0$ to 3

ii) Determine the closed form solution for $y(kT)$ as a function of k

(8 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}(1+4az^{-1}+a^2z^{-2})}{(1-az^{-1})^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3})}{(1-az^{-1})^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$