



TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Applied Physics

EEE 4450: CONTROL III

END OF SEMESTER EXAMINATION

SERIES: NOVEMBER 2024 EXAM

TIME: 2 HOURS

DATE: Pick Date Select Month Pick Year

Instructions to Candidates

You should have the following for this examination

-Answer Booklet, examination pass and student ID

A table of Laplace and Z-transforms

This paper consists of **five** Questions; Question ONE is compulsory. In addition, attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE (Compulsory 20 marks)

- (a) Define state of a system, state variables and state vector. (3 marks)
- (b) A system is described by the following differential equation. Represent the system in state space:
 $\ddot{y} + 3\dot{y} + 4y = 6u_1 + 3u_2 + 4u_3$
 And the outputs are:
 $y_1 = 4\dot{x} + 3u_1$
 $y_2 = \ddot{x} + 4u_2 + u_3$
 Determine the state space representation of the system in diagonal canonical form. (8 marks)
- (c) A feedback system has a closed loop transfer function:
 $\frac{Y(s)}{R(s)} = \frac{10s + 40}{s^3 + s^2 + 3s}$. Construct three different state models showing block diagram in each case. (9 marks)

Question TWO

- (a) A two input two output linear dynamic system is governed by:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Find

i) The Transfer function matrix

ii) Assuming $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ find the output response $y(t)$ if $u(t) = \begin{bmatrix} 0 \\ e^{-3t} \end{bmatrix}$ for $t \geq 0$

(12 marks)

(b) Derive a state space model for the electrical parallel RLC network given in Figure Q3 (b) given that the current $i = I \sin \omega t$.

(8 marks)

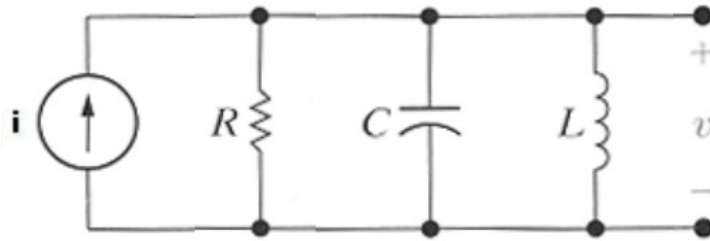


Figure 3 (b)

Question THREE

(a) Distinguish between singularity and non-singularity matrix

(2 marks)

(b) Show the following system is always controllable

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

(4 marks)

(c) Explain the design of state observer.

(4 marks)

(d) Illustrate and explain pole placement by state feedback.

(4 marks)

(e) Find the matrix $k = [k_1 \ k_2]$ which is called the state feedback gain matrix for the closed loop poles to be located at $-1.8 \pm j2.4$ for the original system governed by the state equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(6 marks)

Question FOUR

(a) State any Two advantages and Two disadvantages of Digital Control Systems

(4 marks)

(b) Assume the sampling period is 2 secs, discretize the continuous-time system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

(6 marks)

(d) Find the response of the system of Figure Q 4(d) to a unit impulse input

(4 marks)

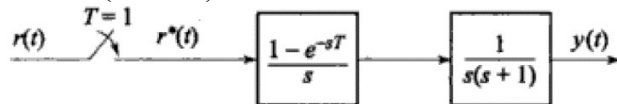


Figure Q 4(d)

(10 marks)

Question FIVE

(a) Derive the transition matrix $\Phi(k)$ of the discrete-time system

$$x(k+1) = Ax(k) + bu(k)$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(5 marks)

(b) Calculate the state vector $x(k)$ for step input.

(5 marks)

(c) Derive the discrete model at the state space of a continuous-time system, which is described by the differential equation:

$\ddot{y}+3\dot{y}+2y=\dot{u}+3u$ and is being discretized with ZOH.

(10 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	—	—	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	—	—	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}(1+4z^{-1}+z^{-2})}{(1-z^{-1})^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{(1-e^{-aT})z^{-1}}{(1-z^{-1})(1-e^{-aT}z^{-1})}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{(e^{-aT}-e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
12.	$\frac{2}{(s+a)^3}$	$t^2 e^{-at}$	$(kT)^2 e^{-akT}$	$\frac{T^2 e^{-aT}(1+e^{-aT}z^{-1})z^{-1}}{(1-e^{-aT}z^{-1})^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{[(aT-1+e^{-aT})+(1-e^{-aT}-aTe^{-aT})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-aT}z^{-1})}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1} \sin \omega T}{1-2z^{-1} \cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1} \cos \omega T}{1-2z^{-1} \cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at} \sin \omega t$	$e^{-akT} \sin \omega kT$	$\frac{e^{-aT} z^{-1} \sin \omega T}{1-2e^{-aT} z^{-1} \cos \omega T+e^{-2aT} z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at} \cos \omega t$	$e^{-akT} \cos \omega kT$	$\frac{1-e^{-aT} z^{-1} \cos \omega T}{1-2e^{-aT} z^{-1} \cos \omega T+e^{-2aT} z^{-2}}$
18.	—	—	a^k	$\frac{1}{1-az^{-1}}$
19.	—	—	a^{k-l} $k = 1, 2, 3, \dots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	—	—	ka^{k-l}	$\frac{z^{-1}}{(1-az^{-1})^2}$
21.	—	—	$k^2 a^{k-l}$	$\frac{z^{-1}(1+az^{-1})}{(1-az^{-1})^3}$
22.	—	—	$k^3 a^{k-l}$	$\frac{z^{-1}(1+4az^{-1}+a^2 z^{-2})}{(1-az^{-1})^4}$
23.	—	—	$k^4 a^{k-l}$	$\frac{z^{-1}(1+11az^{-1}+11a^2 z^{-2}+a^3 z^{-3})}{(1-az^{-1})^5}$
24.	—	—	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
 $x(kT) = x(k) = 0$ for $k < 0$
 Unless otherwise noted, $k = 0, 1, 2, 3, \dots$

