TECHNICAL UNIVERSITY OF MOMBASA

School of Engineering and Technology

Department of Electrical and Electronic Engineering

UNIVERSITY EXAMINATION FOR:

Bachelor of Technology in Applied Physics

EEE 4450: CONTROL III

END OF SEMESTER EXAMINATION

SERIES: AUGUST 2024 EXAM

TIME: 2 HOURS

DATE: Pick DateSelect MonthPick Year

Instructions to Candidates

You should have the following for this examination

-*Answer Booklet, examination pass and student ID*

A table of Laplace and Z-transforms

This paper consists of **five** Questions; Question ONE is compulsory. In addition, attempt any Other TWO Questions.

Do not write on the question paper.

Question ONE (Compulsory 20 marks)

(a) Explain the following terms:

i) State of a dynamic system

ii) State space

(2 marks)

(b) State any TWO advantages of state space approach to control systems analysis. (2 marks)

(c) Given the system with the dynamics

$\ddot{y}+6\ddot{y}+11\{\dot{y}+6y=6u$¿

Determine the state space representation of the system in diagonal canonical form. (9 marks)

(d) Write down the state equation and output equation for the system shown in Figure Q 1(d) (7 marks)

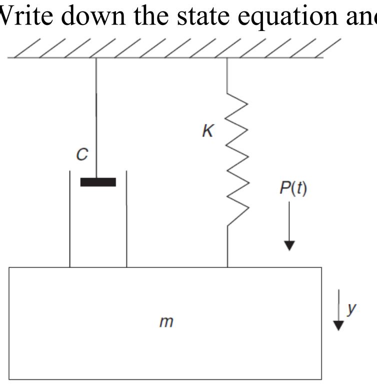

Figure Q 1(d)

Question TWO

(a) A state variable description of a system is given by the matrix equation,

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Find

i) The Transfer function
ii) The State transition matrix
iii) The State and output diagram

(15 marks)

(b) Find the state transition matrix by infinite series method for the system matrix

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

(5 marks)

Question THREE

(a) Distinguish between singularity and non-singularity matrix (2 marks)

(b) State what is meant by Eigen values and Eigen vectors (4 marks)

(c) The state space equation of a control system is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Determine whether the system is completely controllable and observable. (6 marks)

(d) A control system has an open loop transfer function

$$\frac{U(s)}{Y(s)} = \frac{1}{s(s+4)}$$

When $x_1 = y$ and $x_2 = \dot{x}_1$, express the state equation in the controllable canonical form and evaluate the coefficients of the state feedback gain matrix such that the closed loop poles have the values $s=-2, s=-2$

(8 marks)

Question FOUR

(a) State any Two advantages and Two disadvantages of Digital Control Systems (4 marks)

(b) Find the z-transform of the unit step function $f(t)=1$ (4 marks)

(c) A discrete time system is to have the identical time response to the continuous system. State the desired closed loop poles and characteristic equations in

i) the s plane and
ii) the z plane

(4 marks)

(d) For a system illustrated in Figure Q 4(d) with the transfer function $G(s) = \frac{1}{(s+1)(s+2)}$ obtain the pulse transfer function $G(z)$

(8 marks)

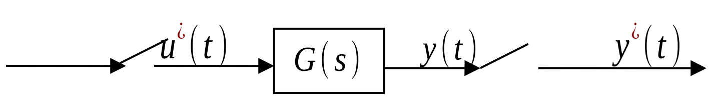

Figure Q 4(c)

Question FIVE

(a) State the following Z- transformation theorems:

i) Linearity
ii) Shift theorem
iii) Final value theorem
i) Initial value theorem

(4 marks)

(b) Derive the state equations and the transfer function $H(z)=Y(z)/R(z)$ for the block diagram of Figure Q5 (b).

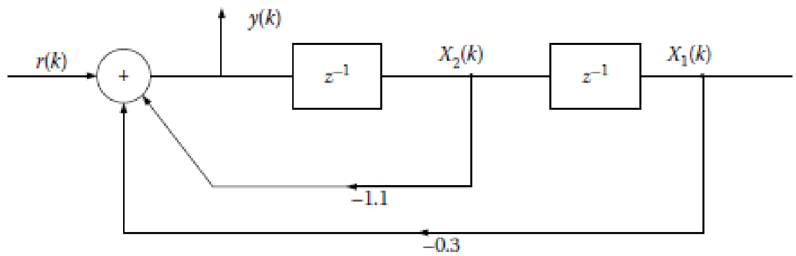

Figure Q5 (b). (8 marks)

(c) Given that:

$U(z)=\frac{z}{(z-2)(z-4)}$, Expand $U(z)$ into partial fraction form and find its time domain representation using long division method. (8 marks)

Table of Laplace and Z-transforms

	$X(s)$	$x(t)$	$x(kT)$ or $x(k)$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k=0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n=k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1(k)$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2 z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3 z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}$
8.	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$1-e^{-akT}$	$\frac{\left(1-e^{-aT}\right)z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-aT}z^{-1}\right)}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$	$e^{-akT}-e^{-bkT}$	$\frac{\left(e^{-aT}-e^{-bT}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)\left(1-e^{-bT}z^{-1}\right)}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
11.	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$	$(1-akT)e^{-akT}$	$\frac{1-(1+aT)e^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}\left(1+e^{-aT}z^{-1}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at-1+e^{-at}$	$akT-1+e^{-akT}$	$\frac{\left[\left(aT-1+e^{-aT}\right)+\left(1-e^{-aT}-aTe^{-aT}\right)z^{-1}\right]z^{-1}}{\left(1-z^{-1}\right)^2\left(1-e^{-aT}z^{-1}\right)}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos \omega T}{1-2z^{-1}\cos \omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos \omega T}{1-2e^{-aT}z^{-1}\cos \omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{\left(1-az^{-1}\right)^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}\left(1+az^{-1}\right)}{\left(1-az^{-1}\right)^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}\left(1+4az^{-1}+a^2z^{-2}\right)}{\left(1-az^{-1}\right)^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}\left(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3}\right)}{\left(1-az^{-1}\right)^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ for $t < 0$
$x(kT) = x(k) = 0$ for $k < 0$
Unless otherwise noted, $k = 0, 1, 2, 3, \ldots$